

Differential Equations II for Engineering Students

Homework sheet 4

Exercise 1: Determine the type of the following partial differential equations

a) $2u_{xx} - 8u_{xy} + 8u_{yy} + u_y = u$,

b) $2u_{xy} + u_{yy} + xu_x = \cos(y)$,

c) $3u_{xx} + 2u_{xy} + u_{yy} = 0$,

d) $u_{xx} + e^x u_{yy} + \sin(x)(u_x + u_y) = y + x$,

e) $(x^2 + y^2)u_{xx} + 2(x + y)u_{xy} + u_{yy} = 0$.

Exercise 2: Given the initial value problem

$$\begin{aligned}u_{xx} - 3u_{xt} - 4u_{tt} &= 0 \quad \text{for } x \in \mathbb{R}, t \in \mathbb{R}^+ \\u(x, 0) &= 0 \quad \text{for } x \in \mathbb{R}, \\u_t(x, 0) &= 2xe^{-x^2}. \quad \text{for } x \in \mathbb{R}.\end{aligned}$$

Solve the problem using substitution $\alpha = x + \frac{t}{4}$, $\mu = x - t$.

Hint: Compute $v_{\alpha\mu}$ for $v(\alpha, \mu) := u(x(\alpha, \mu), t(\alpha, \mu))$.

Alternatively: convert the derivatives in terms of x, t into derivatives in terms of α, μ .

Submission: 02.06.2023