Fachbereich Mathematik der Universität Hamburg **SuSe 2023**
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Differential Equations II for Engineering Students

Exercise sheet 2

Exercise 1: Old exam question

Given the initial value problem

$$\begin{aligned} u_t + 4t\,u_x &= 3\,, & x \in \mathbb{R},\, t \in \mathbb{R}^+,\\ u(x,0) &= \sin(2x) & x \in \mathbb{R}\,. \end{aligned}$$

a) determine the equations of the characteristics and solve them,

b) compute the solution to the initial value problem for $u(x,t)$.

Exercise 2: (For the very fast students)

Solve the Cauchy problem

$$\begin{aligned} u_t - 4e^{-x}u_x &= 1 & x \in \mathbb{R}\,, t > 0\,,\\ u(x,0) &= x & x \in \mathbb{R}\,. \end{aligned}$$

Exercise 3: Old exam question

Given are the following differential equations for $u(x,t)$, $u : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$

A) $u_t + 3u^3\,u_x = 0$,

B) $u_t + 3x\,u_x = 0$,

C) $u_t + 3\,u_x = 1$.

and the initial condition

$u(x,0) = u_0(x), \qquad x \in \mathbb{R}\,,$

where $u_0 : \mathbb{R} \to \mathbb{R}$ is a monotonically increasing and continuously differentiable function.

For which of the differential equations A), B), C) do the following statements i) and or ii) hold for the solution of the associated initial value problem?

i) The solution is constant along the characteristics?
ii) Are the characteristics straight lines?

Justify your answers.

Discussion: 24.04.23 - 28.04.23