Fachbereich Mathematik der Universität Hamburg **SuSe 2023**
Prof. Dr. J. Behrens
Dr. H. P. Kiani, Sofiya Onyshkevych

Differential Equations II for Engineering Students

Work sheet 1

Exercise 1: (Repetition of DGL I)

a) Let λ be any fixed real number. Determine a real representation of the general solution to the differential equation
$$y''(t) - \lambda\, y(t) \;=\; 0\,.$$

b) Let L be another fixed positive real number. Determine all solutions to the boundary value problem
$$y''(t) - \lambda\, y(t) \;=\; 0 \quad y(0) \;=\; y(L) \;= 0\,.$$
For which $\lambda \in \mathbb{R}$ does the boundary value problem have nontrivial solutions?

The λ-values for which there exist non-trivial solutions (i.e. solutions that are not constantly equal to zero) are called eigenvalues of the problem. The corresponding solutions are called eigenfunctions.

Remark: *The solutions to this eigenvalue problem will be needed again and again during the semester!*

Exercise 2) (Repetition Analysis II)

Given $$f(t) = \begin{cases} 4t & t \in [0, \frac{1}{2}] \\ 4 - 4t & t \in [\frac{1}{2}, 1] \\ 0 & t \in [1, 2] \end{cases}$$

a) make a sketch of the direct 2-periodic continuation of f, and even and odd 4-periodic continuation of f

b) compute the real Fourier series of the odd 4–periodic continuation of f,

c) compute the real Fourier series of the even, 4–periodic extension of f.

Recall:

Let $f : \mathbb{R} \to \mathbb{R}$ integrable and **periodic** with period $T > 0$, i.e.
$$f(t + T) \;=\; f(t)\,, \qquad \forall\, t \in \mathbb{R}$$

Define the angular frequency $\omega = \dfrac{2\pi}{T}$ and denote by

$T_n \;:=\; \left\{ g : \mathbb{R} \to \mathbb{R}\,, g(t) = \frac{a_0}{2} + \sum_{k=1}^{n} \left(a_k \cos(k\omega t) + b_k \sin(k\omega t)\right), \, a_k, b_k \in \mathbb{R}, \right\}$ the space of all $T-$ periodic **trigonometric polynomials of degree n** with the

inner product: $\quad < f, g > := \frac{2}{T} \int_0^T f(t) \cdot g(t)\, dt\,.$

and the norm $\quad \| g \| \;:=\; \sqrt{\frac{2}{T} \int_0^T (g(t))^2\, dt} \;=\; \sqrt{< g, g >}\,.$

Then the functions $\left\{ \frac{1}{\sqrt{2}}, \cos(k\omega t), \sin(k\omega t) : \quad k \in \mathbb{N} \right\}$

are an orthonormal system and the **truncated Fourier series of** f

$$f_n(x) := \frac{a_0}{2} + \sum_{k=1}^{n} \left(a_k \cos(k\omega t) + b_k \sin(k\omega t)\right)$$

with

$$a_k := \frac{2}{T} \int_0^T f(t)\, \cos(k\omega t)\, dt\,, \qquad k \in \mathbb{N}_0,$$
$$b_k := \frac{2}{T} \int_0^T f(t)\, \sin(k\omega t)\, dt\,, \qquad k \in \mathbb{N}\,.$$

is the best approximation for f from T_n , i.e.

$$\| f - f_n \| < \| f - g \| \quad \forall g \in T_n\,.$$

If **f even** then $b_k = 0$ and $\quad a_k = \frac{4}{T} \int_0^{T/2} f(t) \cos(k\omega t)\, dt \qquad k \in \mathbb{N}_0\,.$

If **f is odd** then $a_k = 0$ and $\quad b_k = \frac{4}{T} \int_0^{T/2} f(t) \sin(k\omega t)\, dt \qquad k \in \mathbb{N}\,.$

Discussion: As April 10 is a public holiday due to Easter Monday, the English exercise group is cancelled. Please attend one of the German exercise groups:

`https://www.math.uni-hamburg.de/teaching/export/tuhh/cm/d2/23/gruppen.html.de`