Fachbereich Mathematik der Universität Hamburg **SuSe 2023**
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Differential Equations II for Engineering Students

Homework sheet 1

Exercise 1: (Repetition Analysis II)

For the derivation of parameter-dependent integrals for sufficiently smooth f holds the **Leibniz–Rule** :

$$\boxed{\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)\,dt = \int_{a(x)}^{b(x)} \frac{d}{dx} f(x,t)dt + b'(x)\,f(x,b(x)) - a'(x)\,f(x,a(x))}$$

Find the derivative of the function $F(x)$ defined as

$$F(x) := \int_{-x}^{x^2} e^{xt}\,dt$$

and compute $\lim\limits_{x\to 0} F'(x)$.

Exercise 2: (Repetition of Analysis II)

Determine the appropriate real Fourier series for the following functions:

a) Odd $2L-$ periodic continuation of
$f : [0,1[\to \mathbb{R}, \quad f(x) = \sin(4\pi x) + 2\sin(6\pi x) \quad L = 1\,.$

b) Even $2L-$ periodic continuation of
$f : [-\frac{\pi}{4}, \frac{5\pi}{4}[\to \mathbb{R}, \quad L = \pi$ with

$$f(t) = \begin{cases} 2, & -\frac{\pi}{4} \le t < \frac{\pi}{4}, \\ 0, & \frac{\pi}{4} \le t < \frac{3\pi}{4}, \\ 2, & \frac{3\pi}{4} \le t < \frac{5\pi}{4}. \end{cases}$$

Remark: *For DGL II you will need to know how to calculate Fourier series. Please repeat if necessary!*

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