Mathematics Department
Prof. Dr. J. Behrens

WiSe 2023/24

Exam Differential Equations II

04. March 2024

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters each in the designated fields following. These entries will be stored on data carriers.

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I was instructed about the fact that the required test performance will only be assessed if the TUHH examination office can assure my official admission before the exam's beginning.

Signature:

Task no.	Points	Evaluater
1		
2		
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Exercise 1: [7 points]

Consider the following initial value problem for $u(x,t)$:

$$u_t + u \cdot u_x = 0, \qquad x \in \mathbb{R},\ t \in \mathbb{R}^+$$

$$u(x,0) = \begin{cases} 4 & x \leq -1, \\ 0 & -1 < x \leq 0, \\ -4 & 0 < x\,. \end{cases}$$

a) Determine the entropy solution for $t \in [0, t^*)$ with a sufficiently small t^*.

b) Up to which t^* can the solution from a) be continued at most?

c) Determine the entropy solution for $t > t^*$.

Exercise 2: [3 points]

Given is the following differential equation for $u(x, y)$:

$$x \cdot u_{xx} - (x + y)u_{xy} + y \cdot u_{yy} = 0.$$

Provide the order of the differential equation and determine the type of the differential equation (elliptic, parabolic or hyperbolic) at the points
$\binom{x_1}{y_1} = \binom{1}{1}$ and $\binom{x_2}{y_2} = \binom{1}{-1}$.

Exercise 3: [4 points]

Let u be a harmonic function in the disc $\Omega := \{\binom{x}{y} \in \mathbb{R}^2 : x^2 + y^2 < 25\}$ with given values $g(x, y)$ on the boundary of the disc:

$$\begin{aligned} \Delta u(x,y) &= 0 \qquad \text{for } \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 : x^2 + y^2 < 25 \\ u(x,y) &= g(x,y) \qquad \text{for } \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 : x^2 + y^2 = 25. \end{aligned}$$

In the following two cases one can find solutions without long calculations. Give a solution for each case. Justify your answers.

a) $g(x, y) = \dfrac{x + y + 18}{9}$.

b) $g(x, y) = 2x^2 + 2y^2$.

Exercise 4: [6 points]

Determine the solution to the initial boundary value problem

$$\begin{aligned}
u_{tt} - 36u_{xx} &= 0 && 0 < x < 2\pi\,,\ 0 < t\,,\\
u(x,0) &= 20\sin(\tfrac{3}{2}x) && 0 \le x \le 2\pi,\\
u_t(x,0) &= 24\sin(3x) && 0 \le x \le 2\pi,\\
u(0,t) &= 0 && 0 \le t,\\
u(2\pi,t) &= 0 && 0 \le t.
\end{aligned}$$