Mathematics Department
Prof. Dr. J. Behrens

SuSe 2023

Exam Differential Equations II

05. September 2023

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters each in the designated fields following. These entries will be stored on data carriers.

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I was instructed about the fact that the required test performance will only be assessed if the TUHH examination office can assure my official admission before the exam's beginning.

Signature:

Task no.	Points	Evaluater
1		
2		
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Exercise 1: [5 points]

Compute the solution to the following initial value problem for $u(x,t)$:

$$
\begin{aligned}
u_t - 4t^3 u_x &= e^{-t}, \qquad && x \in \mathbb{R},\, t \in \mathbb{R}^+, \\
u(x,0) &= 1 + \sin(2x) && x \in \mathbb{R}.
\end{aligned}
$$

Exercise 2: [4= 2+2 points]

a) The functions

$$u_1(x,t) = \begin{cases} 0 & x \le 1+t \\ 2 & x > 1+t \end{cases} \qquad \text{and} \qquad u_2(x,t) = \begin{cases} 0 & x \le 1 \\ \frac{x-1}{t} & 1 < x < 2t+1 \\ 2 & 2t+1 \le x \end{cases}$$

are both weak solutions to the following initial value problem

$$u_t + \left(\frac{u^2}{2}\right)_x = u_t + u \cdot u_x = 0, \qquad u(x,0) = \begin{cases} 0 & x \le 1, \\ 2 & x > 1 \end{cases} \qquad x \in \mathbb{R},\ t \in \mathbb{R}^+.$$

Determine and justify which of the two weak solutions is the entropy solution.

b) Given is the initial value problem

$$v_t + \left(\frac{v^2}{2}\right)_x = v_t + v \cdot v_x = 0, \qquad v(x,0) = \begin{cases} 2 & x \le 0, \\ -2 & x > 0. \end{cases}$$

Determine and justify which of the following functions v_1 or v_2

$$v_1(x,t) = \begin{cases} 2 & x \le 0 \\ -2 & x > 0 \end{cases} \qquad v_2(x,t) = \begin{cases} 2 & x \le 2t \\ -2 & x > 2t \end{cases}$$

is the entropy solution to the initial value problem.

Exercise 3: [1+1+2 points]

Given the differential equation

$$4u_{xx} - 2u_{xt} + 4u_{tt} \;=\; 0 \quad \text{for } x \in \mathbb{R}, t > 0$$

a) rewrite the differential equation in matrix notation.

b) determine the type of the differential equation (elliptic, hyperbolic or parabolic).

c) transform the differential equation to a diagonal form $\alpha \cdot \tilde{u}_{\eta\eta} + \beta \cdot \tilde{u}_{\tau\tau} = 0\,.$

Exercise 4: [6+1 point]

a) Given is the initial boundary value problem

$$
\begin{aligned}
u_{tt} - 9u_{xx} &= \left(\frac{x}{2} - 1\right)\sin(t) && \text{for } x \in (0,2), t > 0,\\
u(x,0) &= \frac{x}{4} && \text{for } x \in [0,2],\\
u_t(x,0) &= 1 - \frac{x}{2} && \text{for } x \in [0,2],\\
u(0,t) &= \sin(t), \quad u(2,t) = \frac{1}{2} && \text{for } t > 0.
\end{aligned}
$$

Using a suitable homogenization of the boundary data, transform the problem into an initial boundary value problem with homogeneous boundary data for a function $v(x,t)$.

Provide the new initial boundary value problem (differential equation, initial conditions and boundary conditions).

b) Determine the solution to the initial boundary value problem for $v(x,t)$ from a).