Fachbereich Mathematik der Universität Hamburg **SuSe 2022**
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Differential Equations II for Engineering Students

Homework sheet 2

Exercise 1: [5 Points]

Compute the solution to the following initial value problem for $u(x,t)$:

$$u_t - \sin(t)\, u_x = \cos(t)\,, \qquad x \in \mathbb{R},\ t \in \mathbb{R}^+,$$
$$u(x,0) = \exp(-x^2) = e^{-x^2} \qquad x \in \mathbb{R}\,.$$

Exercise 2: [6= 2+1+2+1 points]

Given are the following differential equations for $u(x,t)$, $u : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$

A) $u_t + 20\, u_x = 21u\,.$

B) $u_t + 20u\, u_x = 21\,.$

C) $u_t - 5u^2\, u_x = 0.$

D) $u_t + 5(x+1)\, u_x = 0.$

with the initial condition

$u(x,0) = u_0(x), \qquad x \in \mathbb{R}\,,$

where $u_0 : \mathbb{R} \to \mathbb{R}$ is a monotonically increasing and continuously differentiable function.

For which of the differential equations A, B, C, D do the following statements i) and/or ii) hold for the solution of the associated initial value problem?

i) The solution is constant along the characteristics.
ii) The characteristics are straight lines.

Explain your answers. Note that you don't have to compute any solutions!

Exercise 3:

Determine a continuous "solution" $u(x,t)$ to the following initial boundary value problem

$$u_t + u_x = x, \qquad x,t > 0$$
$$u(x,0) = x \qquad (x \geq 0)$$
$$u(0,t) = t \qquad (t \geq 0)$$

using the method of characteristics. To do this, determine the solution to the initial condition $u(x,0) = x$ and to the boundary condition $u(0,t) = t$ and continuously compose these solutions. Is the solution obtained in this way partially differentiable for all $x,t \geq 0$?

Voluntary additional task: If you like, you can do the task too using the Laplace transformation with respect to the variable t. For the transformation x is used as a parameter. In the image space, an initial value problem and an ordinary differential equation are to be solved with respect to for x.

Submission deadline: 25.04.-29.04.2022