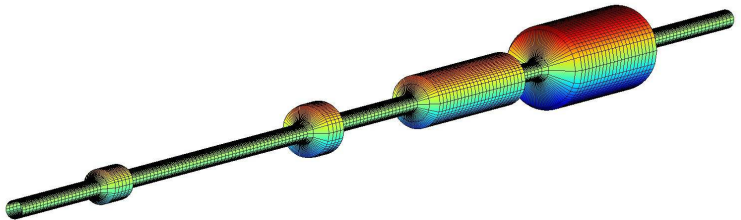


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Modeling, Simulation and Optimization of Gas Dynamics in an Exhaust Pipe: A Network Approach



10.12.2012

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Exhaust gas pollution

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Figure: exhaust gas emission¹ and air pollution in China²

¹<http://www.thedailygreen.com/cm/thedailygreen/images/o6/car-exhaust-lg.jpg>

²<http://climatechange.foreignpolicyblogs.com/files/2010/09/china-air-pollution.jpg>

Catalytic converter

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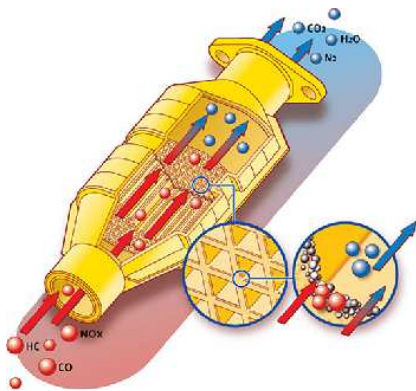


Figure: catalytic converter³

- Catalytic converters reduces needed activation energy for the transformation of harmful gases into less harmful gases
- Honeycomb structure provides a large reaction surface
- Reaction only happens when activation energy $\tilde{E}^+ \approx 600\text{K}$ is reached ("light off")

³

http://www.timbarkerstudio.com/technical/catalytic_converter.jpg

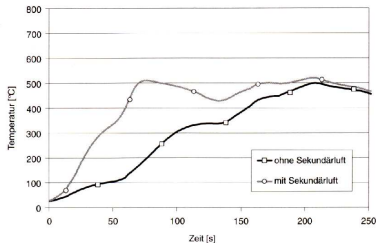
Catalytic converter II

How to ensure sufficient high temperatures in the catalytic converter just after the engine start?

- Choose λ -ratio less than 1.

$$\lambda = \frac{m_{air}}{m_{air}^{st}}$$

- Unburnt gas is transported to the catalytic converter, where it reacts exothermically
- Required oxygen is given by a pump



R. van Basshuysen
and F. Schäfer
Handbuch
Verbrennungsmotor
Vieweg 2005 ,page
711

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Modeling

- Gas flow through a pipe with a variable duct
- physical effects have to be included, such as
 - wall friction
 - heat transfer through the wall
 - combustion in the catalytic converters
 - friction in the catalytic converters due to honeycomb structure

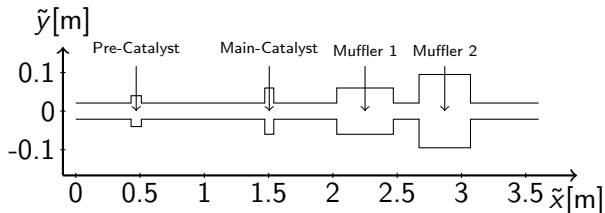


Figure: Geometry of the exhaust pipe

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First model

$$(\tilde{A}\tilde{\rho})_{\tilde{t}} + (\tilde{A}\tilde{\rho}\tilde{u})_{\tilde{x}} = 0$$

$$(\tilde{A}\tilde{\rho}\tilde{u})_{\tilde{t}} + (\tilde{A}\tilde{\rho}\tilde{u}^2)_{\tilde{x}} + \tilde{A}\tilde{p}_{\tilde{x}} = -\frac{\xi}{4}\pi\tilde{d}\tilde{\rho}\frac{\tilde{u}^2}{2} - \tilde{C}_c\tilde{A}\tilde{\chi}\tilde{\rho}\tilde{u}$$

$$(\tilde{A}\tilde{\rho}\tilde{E})_{\tilde{t}} + (\tilde{A}\tilde{\rho}\tilde{u}\tilde{E} + \tilde{A}\tilde{u}\tilde{p})_{\tilde{x}} = -\tilde{h}\pi\tilde{d}(\tilde{T} - \tilde{T}_{\text{Wall}}) \quad (1)$$
$$+ \tilde{q}_0\tilde{A}\tilde{\chi}\tilde{\rho}\tilde{z}\tilde{K}(\tilde{T})$$

$$(\tilde{A}\tilde{\rho}\tilde{z})_{\tilde{t}} + (\tilde{A}\tilde{\rho}\tilde{u}\tilde{z})_{\tilde{x}} = -\tilde{A}\tilde{\chi}\tilde{\rho}\tilde{z}\tilde{K}(\tilde{T})$$

$$\tilde{p} = R\tilde{\rho}\tilde{T}$$

with $\tilde{E} = c_v\tilde{T} + \frac{\tilde{u}^2}{2}$, $\tilde{K}(\tilde{T}) := \tilde{K}_0 \cdot \exp(-\tilde{E}^+/\tilde{T})$ (Arrhenius' law) and $\tilde{\chi}(x) = 1 \Leftrightarrow x$ is in one of the two catalytic converters, and vanishes otherwise.



R. Natalini and L. Lacoste for MAGNETTI MARELLI

Mathematical modeling of chemical processes in exhaust pipe

Technical Report, IAC Rome, 2004.

New model

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Model for a network of single pipes.

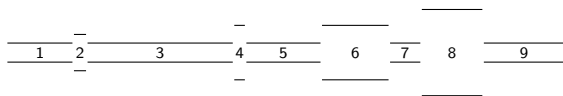


Figure: Network of 9 connected pipes

Use model (1) with constant cross section to derive a model for the single pipes.

New model II

Model for a single pipe with diameter \tilde{d} containing a catalytic converter.

$$\tilde{\rho}\tilde{z} + (\tilde{\rho}\tilde{u})_{\tilde{x}} = 0$$

$$(\tilde{\rho}\tilde{u})_{\tilde{t}} + (\tilde{\rho}\tilde{u}^2 + \tilde{p})_{\tilde{x}} = -\frac{\xi}{\tilde{d}}\tilde{\rho}\frac{\tilde{u}^2}{2} - \tilde{C}_c\tilde{\rho}\tilde{u}$$

$$(\tilde{\rho}\tilde{E})_{\tilde{t}} + (\tilde{\rho}\tilde{u}\tilde{E} + \tilde{u}\tilde{p})_{\tilde{x}} = -\frac{4\tilde{h}}{\tilde{d}}(\tilde{T} - \tilde{T}_{\text{Wall}}) + \tilde{q}_0\tilde{\rho}\tilde{z}\tilde{K}(\tilde{T}) \quad (2)$$

$$(\tilde{\rho}\tilde{z})_{\tilde{t}} + (\tilde{\rho}\tilde{u}\tilde{z})_{\tilde{x}} = -\tilde{\rho}\tilde{z}\tilde{K}(\tilde{T})$$

$$\tilde{p} = R\tilde{\rho}\tilde{T}$$

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New model II

Model for a single pipe with diameter \tilde{d} containing a catalytic converter.

$$\tilde{\rho}_{\tilde{t}} + (\tilde{\rho}\tilde{u})_{\tilde{x}} = 0$$

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$$(\tilde{\rho}\tilde{E})_{\tilde{t}} + (\tilde{\rho}\tilde{u}\tilde{E} + \tilde{u}\tilde{p})_{\tilde{x}} = -\frac{4\tilde{h}}{\tilde{d}}(\tilde{T} - \tilde{T}_{\text{Wall}}) + \tilde{q}_0\tilde{\rho}\tilde{z}\tilde{K}(\tilde{T}) \quad (2)$$

$$(\tilde{\rho}\tilde{z})_{\tilde{t}} + (\tilde{\rho}\tilde{u}\tilde{z})_{\tilde{x}} = -\tilde{\rho}\tilde{z}\tilde{K}(\tilde{T})$$

$$\tilde{p} = R\tilde{\rho}\tilde{T}$$

Simplify this model:

- 1 Scale
- 2 Small Mach number \rightarrow asymptotic model

Scaling

Replace each quantity (\tilde{y}) by the product of the reference quantity (y_r) and the dimensionless quantity (y).

| Quantity | Unit | Reference quantity | Reference value |
|----------------|-----------------------------------|-----------------------|-----------------------------------------|
| \tilde{t} | s | $t_r = x_r/u_r$ | 0.01 – 0.093s |
| \tilde{x} | m | $x_r = L$ | 0.1 – 0.93m |
| $\tilde{\rho}$ | kg m^{-3} | ρ_r | 1.2 kg m^{-3} |
| \tilde{u} | m s^{-1} | u_r | 10 m s^{-1} |
| \tilde{p} | $\text{kg m}^{-1} \text{ s}^{-2}$ | p_r | $10^5 \text{ kg m}^{-1} \text{ s}^{-2}$ |
| \tilde{T} | K | $T_r = p_r/(R\rho_r)$ | 300 K |
| \tilde{z} | | z_r | 0.1 |

Table: Quantities, Units, Reference quantities, Reference values

Full Euler model (FE)

Dimensionless model, full Euler model (FE)

$$\begin{aligned}\rho_t + (\rho u)_x &= 0 \\ (\rho u)_t + (\rho u^2)_x + \left(\frac{1}{\gamma M^2}\right) p_x &= -C_f \rho \frac{u^2}{2} - C_c \rho u \\ (\rho T + (\gamma - 1)\gamma M^2 \rho \frac{u^2}{2})_t + (\rho u T + (\gamma - 1)\gamma M^2 \rho \frac{u^3}{2})_x \\ &+ (\gamma - 1)(u p)_x = -h(T - T_{\text{Wall}}) + q_0 \rho z K(T) \\ (\rho z)_t + (\rho u z)_x &= -\rho z K(T) \\ p &= \rho T\end{aligned}$$

(FE)

$$M = u_r \sqrt{\frac{\rho_r}{\gamma p_r}} \approx 3 \cdot 10^{-2}, \quad \left(\frac{1}{\gamma M^2}\right) \approx 10^3, \quad (\gamma - 1)\gamma M^2 \approx 4.8 \cdot 10^{-4}$$

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Asymptotic model (AM)

Asymptotic expansion in $\varepsilon = \gamma M^2$ in the quantities, i.e.

$$p(x, t) = p_0(x) + \varepsilon p_1(x, t) + \mathcal{O}(\varepsilon^2)$$

leads to an asymptotic model.

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Asymptotic model (AM)

Asymptotic expansion in $\varepsilon = \gamma M^2$ in the quantities, i.e.

$$p(x, t) = p_0(x) + \varepsilon p_1(x, t) + \mathcal{O}(\varepsilon^2)$$

leads to an asymptotic model.

$$\rho_t + (v + Q)\rho_x = -q\rho$$

$$z_t + (v + Q)z_x = -zK(T)$$

$$v_t = \frac{1}{\int_0^1 \rho dx} \left[p_l - p_r - \int_0^1 \rho Q_t dx - \int_0^1 \rho(v + Q)q dx - C_f \int_0^1 \frac{\rho(v + Q)^2}{2} dx - C_c \int_0^1 \rho(v + Q) dx \right]$$

(AM)

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The exhaust pipe as a network

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- Each pipe ($i = 1, \dots, 9$) needs 4 boundary conditions (data we do not have).
- Connect 9 pipes to a network by defining **coupling conditions** at the vertices

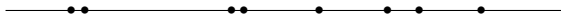


Figure: Network of 9 connected pipes and 8 vertices

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- 1 Conservation of mass

$$\rho_r^{i-1} = \rho_l^i \quad \forall i = 2, \dots, 9 \quad (3)$$

- 2 Conservation of the ratio of unburnt gas

$$z_r^{i-1} = z_l^i \quad \forall i = 2, \dots, 9 \quad (4)$$

- 3 Conservation of internal energy (temperature)

$$u_r^{i-1} A^{i-1} = u_l^i A^i \quad \forall i = 2, \dots, 9 \quad (5)$$

Coupling conditions for the pressure

- 1 First idea: Conservation of pressure at the vertices



$$p_r^i = p_V^i = p_l^{i+1} \quad (6)$$

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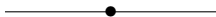
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Coupling conditions for the pressure

- 1 First idea: ~~Conservation of pressure at the vertices~~

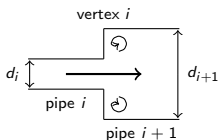


$$p_r^i = p_V^i = p_l^{i+1} + f_{\text{ext}}^i \quad (6)$$

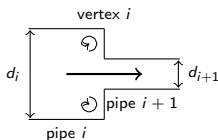
- 2 Include pressure loss term (*minor losses*) at the vertices.

$$f_{\text{ext}}^i = \begin{cases} \rho_l^i \frac{(u_r^i)^2}{2} \left(1 - \frac{d_i^2}{d_{i+1}^2}\right)^2 & \text{sudden expansion} \\ 0.5 \cdot \rho_l^{i+1} \frac{(u_l^{i+1})^2}{2} \left(1 - \frac{d_{i+1}^2}{d_i^2}\right)^2 & \text{sudden contraction} \end{cases}$$

sudden expansion



sudden contraction



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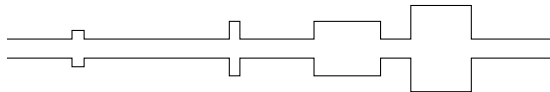
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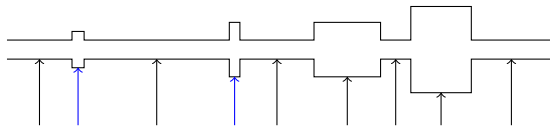
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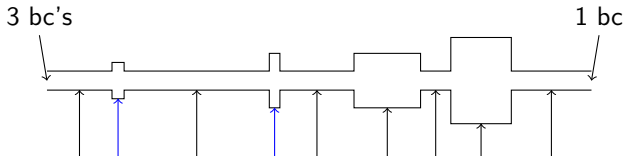
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$$\begin{aligned}\rho_t + (v + Q)\rho_x &= -q\rho \\ z_t + (v + Q)z_x &= -zK(T)\end{aligned}$$
$$v_t = \frac{1}{\int_0^1 \rho dx} \left[p_l - p_r - \int_0^1 \rho Q_t dx - \int_0^1 \rho(v + Q)q dx \right. \\ \left. - C_f \int_0^1 \frac{\rho(v + Q)^2}{2} dx - C_c \int_0^1 \rho(v + Q) dx \right]$$

Overview

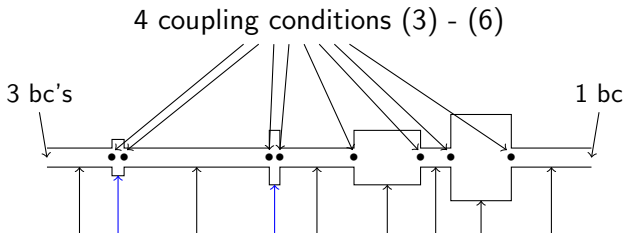
Initial (engine start) and boundary conditions



$$\begin{aligned}\rho_t + (v + Q)\rho_x &= -q\rho \\ z_t + (v + Q)z_x &= -zK(T)\end{aligned}$$
$$v_t = \frac{1}{\int_0^1 \rho dx} \left[p_l - p_r - \int_0^1 \rho Q_t dx - \int_0^1 \rho(v + Q)q dx \right. \\ \left. - C_f \int_0^1 \frac{\rho(v + Q)^2}{2} dx - C_c \int_0^1 \rho(v + Q) dx \right]$$

Overview

Initial (engine start) and boundary conditions



$$\begin{aligned}\rho_t + (v + Q)\rho_x &= -q\rho \\ z_t + (v + Q)z_x &= -zK(T)\end{aligned}$$
$$v_t = \frac{1}{\int_0^1 \rho dx} \left[p_l - p_r - \int_0^1 \rho Q_t dx - \int_0^1 \rho(v + Q)q dx \right. \\ \left. - C_f \int_0^1 \frac{\rho(v + Q)^2}{2} dx - C_c \int_0^1 \rho(v + Q) dx \right]$$

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Numerical costs

CFL-condition for (FE)

$$\lambda_{max} \frac{\Delta t}{\Delta x} \leq c_N$$

with $\lambda_{max} := \max |\lambda|$

CFL-condition for (AM)

$$u_{max} \frac{\Delta t}{\Delta x} \leq c_N$$

with $u_{max} := \max |u|$

c_N denotes the Courant number. $u \geq 0 \Rightarrow \lambda_{max} = u_{max} + c$, where c denotes speed of sound. Then

$$\Delta t_{AM} = \frac{\lambda_{max}}{u_{max}} \Delta t_{FE} = \left(1 + \frac{1}{M}\right) \Delta t_{FE} \quad (7)$$

Furthermore the computation of one spatial steps for the full Euler model takes approximately 5 times as many *flops* as for the asymptotic model.

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We will compare the **full Euler model (FE)** with the **asymptotic model (AM)**.

- Single pipe
 - high pressure difference
 - low pressure difference
- Whole exhaust pipe
 - high pressure difference
 - low pressure difference
- Sensibility numerical solutions to the minor loss term

Single pipe: Example 1

(FE)

(AM)

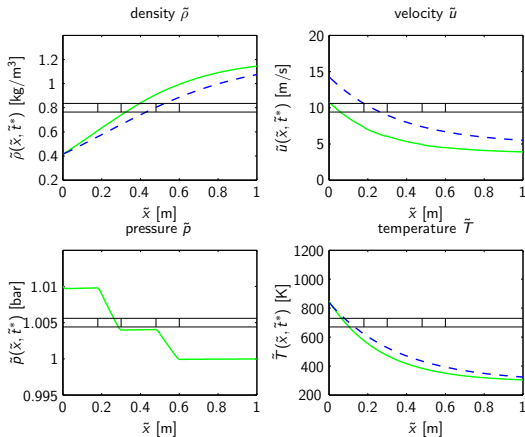


Figure: Example 1: Numerical results after $\tilde{t}^* = 0.4s$

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Single pipe: Example 2

(FE)

(AM)

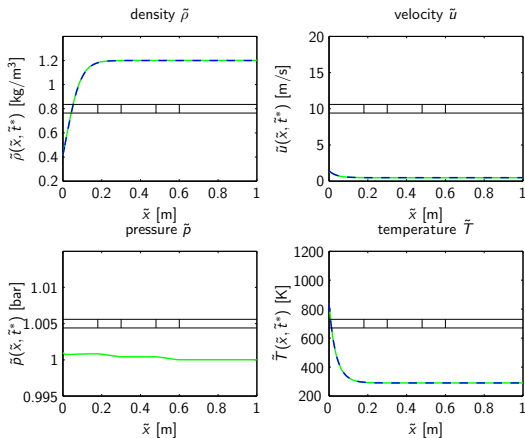


Figure: Example 2: Numerical results after $\tilde{t}^* = 3s$

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Single pipe: Output data

| | Example 1 | | Example 2 | |
|----------------------------------------------------|------------|------------|------------------|------------|
| | FE | AM | FE | AM |
| spatial supporting points | 100 | 100 | 100 | 100 |
| time steps | 25947 | 617 | 176644 | 488 |
| last time step size | 1.5324e-05 | 0.00062999 | 1.6902e-05 | 0.0062657 |
| computation time (in sec) | 113.5494 | 0.29539 | 772.1798 | 0.24294 |
| computation time (in min) | 1.8925 | 0.0049232 | 12.8697 | 0.004049 |
| largest speed ($\lambda_{max}, \tilde{u}_{max}$) | 587.3086 | 4.2859 | 532.4936 | 1.4364 |
| Mach number M | 0.01828 | 0.02459 | 0.0022544 | 0.0024725 |
| ratio FE/AM time steps | 42.0535 | | 361.9754 | |
| ratio AM/FE time steps size | 41.1111 | | 370.7164 | |
| ratio $\lambda_{max} / \tilde{u}_{max}$ | 41.1111 | | 370.7164 | |
| ratio FE/AM computation time | 384.4039 | | 3178.4926 | |

Table: Output data for example 1 and example 2. The computation time refers to a personal computer (Intel(R) Core(TM) i5 CPU 750 @ 2.67GHz) with 8 GB of memory

Whole exhaust pipe: Example 3

(FE) $c_N = 0.1$

(AM) $c_N = 0.1$

(AM) $c_N = 1$

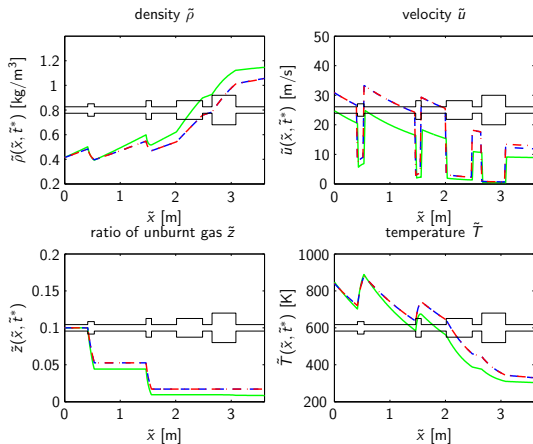


Figure: Example 3: Numerical results after $\tilde{t}^* = 2s$

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Whole exhaust pipe: Example 4

(FE) $c_N = 0.1$

(AM) $c_N = 0.1$

(AM) $c_N = 1$

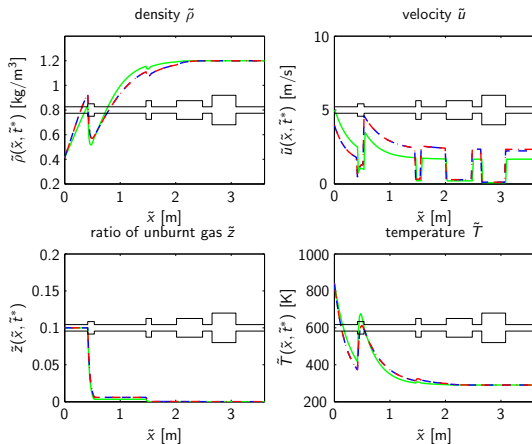


Figure: Example 4: Numerical results after $\tilde{t}^* = 3s$

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Whole exhaust pipe: Output data

(FE) $c_N = 0.1$

(AM) $c_N = 0.1$

(AM) $c_N = 1$

| | Example 3 | | | Example 4 | | |
|------------------------------------------------|-----------|----------|------------|---------------|----------|------------|
| | FE | AM | | FE | AM | |
| spatial supp. points | 200 | 200 | (200) | 200 | 200 | (200) |
| time steps | 392628 | 34052 | (3469) | 570800 | 6849 | (725) |
| last time step size | 4.98e-06 | 5.40e-05 | (5.40e-04) | 5.25e-06 | 3.91e-04 | (3.91e-03) |
| comp. time (in sec) | 5935.86 | 43.97 | (4.75) | 8587.31 | 9.96 | (1.26) |
| comp. time (in min) | 98.93 | 0.73 | (0.08) | 143.12 | 0.17 | (0.02) |
| max speed ($\lambda_{max}, \tilde{u}_{max}$) | 362.02 | 33.32 | (33.28) | 343.24 | 4.61 | (4.61) |
| Mach number M | 0.042 | 0.056 | (0.056) | 0.009 | 0.009 | (0.009) |
| ratio FE/AM time steps | | 11.53 | (113.18) | | 83.34 | (787.31) |
| ratio AM/FE Δt | | 10.83 | (108.37) | | 74.34 | (743.24) |
| ratio $\lambda_{max}/\tilde{u}_{max}$ | | 10.86 | (10.88) | | 74.51 | (74.53) |
| ratio FE/AM comp. time | | 134.98 | (1238.80) | | 861.58 | (6736.72) |

Table: Output data for example 3 and example 4. The computation time refers to a personal computer (Intel(R) Core(TM) i5 CPU 750 @ 2.67GHz) with 8 GB of memory

Minor loss term: Example 5

$$\theta = \pi$$

$$\theta = \pi/3$$

no pressure loss

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Minor loss term: Example 5

$$\theta = \pi$$

$$\theta = \pi/3$$

no pressure loss

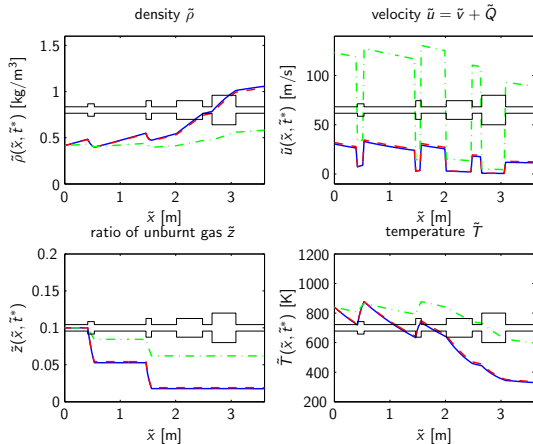


Figure: Example 5: Numerical results after $\tilde{t}^* = 2s$

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Related Work

- accepted and to appear:



I. Gasser and M. Rybicki

Modelling and Simulation of Gas Dynamics in an Exhaust Pipe
Applied Mathematical Modelling, 2012.

- related work



I. Gasser and J. Struckmeier

An asymptotic-induced one-dimensional model to describe Fires in Tunnels
Mathematical methods in the applied sciences (25), p. 1231-1249, 2002.



I. Gasser and M. Kraft

Modelling and simulation of fires in tunnels
Networks in heterogeneous media (3), p. 691-707, 2008.



I. Gasser

Modelling and Simulation of a Solar Updraft Tower
Kinetic and Related Models (2), p. 191-204, 2009.



M. Bauer and I. Gasser

Modelling, Asymptotic Analysis, and Simulation of an Energy Tower
SIAM J. Appl. Math. (72), p. 362-381, 2012.



E. Felaco and I. Gasser

Modelling, Asymptotic Analysis and Simulation of the Gas Dynamics in a Chimney
submitted 2012

- planed



I. Gasser, M. Rybicki and W. Wollner

Optimal Control of Gas Temperature on Networks

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Optimization Problem

- **Aim:** "Heat up CC without using too much fuel".

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- **Aim:** "Heat up CC without using too much fuel".
- **Equation for temperature in catalytic converter** required:

$$(T_c)_t = h_c(T_{gas} - T_c^i) \quad (8)$$

where T_{gas} is the mean gas temperature in the cc.

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$$\mathcal{J} := \frac{1}{2} \sum_{i \in C} \int_0^{t_{end}} (T_c^i(t) - T_{opt})^2 dt + \frac{\sigma}{2} \int_0^{t_{end}} z_l(t)^2 dt \quad (9)$$

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- **Task:** Minimize Cost functional subject to the constraints.

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- **Task:** Minimize Cost functional subject to the constraints.
- **Strategy:** First variations of Lagrange functional lead to an optimality system (KKT-condition)

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Lagrangian functional

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$$\begin{aligned}
 \mathcal{L} = & \frac{1}{2} \sum_{i \in C} \int_0^{t_{\text{end}}} (T_c^i(t) - T_{\text{opt}})^2 dt + \frac{\sigma}{2} \int_0^{t_{\text{end}}} z_1^1(t)^2 dt \\
 & - \sum_{i=1}^{n_P} \int_0^{t_{\text{end}}} \int_0^{L^i} \xi_1^i (\rho_t^i + (v^i + Q^i) \rho_x^i + q^i \rho^i) dx dt - \sum_{i=1}^{n_P} \int_0^{t_{\text{end}}} \int_0^{L^i} \xi_2^i (z_t^i + (v^i + Q^i) z_x^i + z^i K(T^i)) dx dt \\
 & - \sum_{i=1}^{n_P} \int_0^{t_{\text{end}}} \xi_3^i \left(v_t^i - \frac{1}{R^i(0)} \left[p_{1l}^i - p_{1r}^i - \int_0^{L^i} \rho^i Q_t^i + \rho^i (v^i + Q^i) q^i + C_f \frac{\rho^i (v^i + Q^i)^2}{2} dx \right. \right. \\
 & \quad \left. \left. - C_c \int_0^{L^i} \rho^i (v^i + Q^i) dx \right] \right) dt - \sum_{i \in C} \int_0^{t_{\text{end}}} \xi_4^i ((T_c^i)_t - h_c (T_{\text{gas}}^i - T_c^i)) dt \\
 & - \int_0^{t_{\text{end}}} \eta_1 (\rho^1(0, t) - \rho_l(t)) dt - \int_0^{t_{\text{end}}} \eta_2 (z^1(0, t) - z_l(t)) dt - \sum_{i=1}^{n_P} \int_0^{L^i} \nu_1^i (\rho^i(x, 0) - \rho_{ic}^i(x)) dx \\
 & - \sum_{i=1}^{n_P} \int_0^{L^i} \nu_2^i (z^i(x, 0) - z_{ic}^i(x)) dx - \sum_{i=1}^{n_P} \nu_3^i (v^i(0) - v_{ic}^i) - \sum_{i=1}^{n_P} \nu_4^i (T_c^i(0) - T_{cic}^i)
 \end{aligned}$$

Optimality system

- constraints or state equations

$$\begin{aligned}\rho_t^i + (v^i + Q^i)\rho_x^i &= -q^i \rho^i \\ z_t^i + (v^i + Q^i)z_x^i &= -z^i K(T^i) \\ v_t^i &= \Phi^i / R^i(0) \\ (T_c^i)_t &= h_c(T_{\text{gas}}^i - T_c^i)\end{aligned}\tag{10}$$

- adjoint or co-state equations

$$\begin{aligned}(\xi_1^i)_t + (v^i + Q^i)(\xi_1^i)_x &= \dots \\ (\xi_2^i)_t + (v^i + Q^i)(\xi_2^i)_x &= \dots \\ (\xi_3^i)_t &= \dots \\ (\xi_4^i)_t &= \dots\end{aligned}\tag{11}$$

- optimality condition

$$\sigma z_1 + \eta_2 = 0\tag{12}$$

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Algorithm

Guess a initial control $z_l^{(0)}$. For $k = 0, 1, 2, \dots$ repeat the following steps until satisfactory convergence is achieved:

- 1 solve the constraints with control $z_l^{(k)}$ to obtain the corresponding state variables $\rho^{(k)} = \rho(z_l^{(k)})$, $z^{(k)} = z(z_l^{(k)})$, $v^{(k)} = v(z_l^{(k)})$, $T_c^{(k)} = T_c(z_l^{(k)})$;
- 2 solve the adjoint system with state variables $\rho^{(k)}$, $z^{(k)}$, $v^{(k)}$, $T_c^{(k)}$ to obtain the adjoint variables $\xi_1^{(k)}$, $\xi_2^{(k)}$, $\xi_3^{(k)}$, $\xi_4^{(k)}$, $\eta_2^{(k)}$;
- 3 use $\eta_2^{(k)}$ to compute gradient $j'(z_l^{(k)})$;
- 4 compute step length α via line search (ARMIGO or WOLFE-POWELL method);
- 5 set $z_l^{(k+1)} = z_l^{(k)} - \alpha j'(z_l^{(k)})$.

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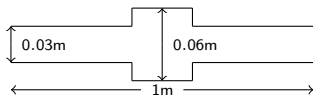
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2 numerical examples

Setting:



$$T_{opt} = 600K$$

$$t_{end} = 60s$$

Examples:

- 1 low costs for boundary control variable, low starting value.

$$\sigma = 0.1 \quad z_l^{(0)}(t) = 0.01 \quad \forall t \in [0, t_{end}]$$

- 2 high costs for boundary control variable, high starting value.

$$\sigma = 1 \quad z_l^{(0)}(t) = 0.1 \quad \forall t \in [0, t_{end}]$$

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Numerical simulations: Example 1

| iteration | $\mathcal{J} = \sigma \mathcal{J}_z + \mathcal{J}_{T_c^2}$ | $\mathcal{J}_z = \frac{1}{2} \int_0^{t_{end}} z_i(t)^2 dt$ | $\mathcal{J}_{T_c^2} = \frac{1}{2} \int_0^{t_{end}} (T_c^2(t) - T_{opt})^2 dt$ |
|-----------|------------------------------------------------------------|------------------------------------------------------------|--------------------------------------------------------------------------------|
| 0 | 118.873 | 3.000 | 118.574 |
| 1 | 69.106 | 31.863 | 65.920 |
| 2 | 47.542 | 73.588 | 40.183 |
| 9 | 35.306 | 171.433 | 18.162 |

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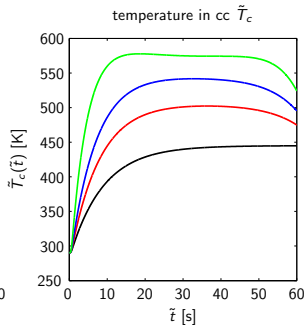
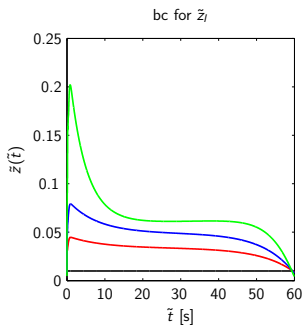
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Numerical simulations: Example 2

| iteration | $\mathcal{J} = \sigma \mathcal{J}_z + \mathcal{J}_{T_c^2}$ | $\mathcal{J}_z = \frac{1}{2} \int_0^{t_{end}} z_l(t)^2 dt$ | $\mathcal{J}_{T_c^2} = \frac{1}{2} \int_0^{t_{end}} (T_c^2(t) - T_{opt})^2 dt$ |
|-----------|------------------------------------------------------------|------------------------------------------------------------|--------------------------------------------------------------------------------|
| 0 | 328.412 | 300.000 | 28.412 |
| 1 | 111.408 | 65.232 | 46.177 |
| 2 | 98.437 | 29.959 | 68.478 |
| 3 | 98.303 | 27.405 | 70.898 |

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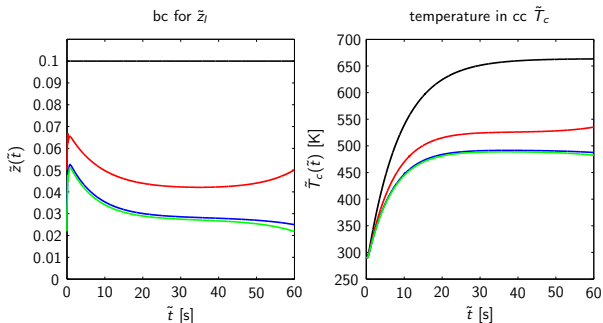
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- Inclusion of pressure loss terms possible with network approach
- very good qualitative consistence of the asymptotic model with the full Euler model.
- Much faster numerical simulations.
- Optimization or control (e.g. of temperature) with respect to boundary conditions possible.

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- Inclusion of pressure loss terms possible with network approach
- very good qualitative consistence of the asymptotic model with the full Euler model.
- Much faster numerical simulations.
- Optimization or control (e.g. of temperature) with respect to boundary conditions possible.

2 Open Questions

- existence of a solution to the constraints
- convergence of the optimization algorithm

Android App

Real-time simulations on a Smartphone, available for download on website of Martin Rybicki:

<http://www.math.uni-hamburg.de/home/rybicki/apps>

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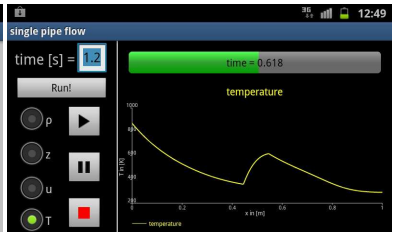
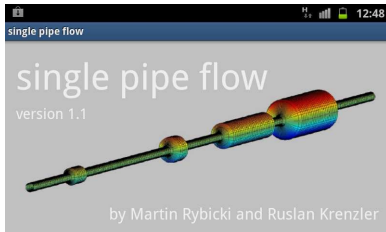
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Thank you!