

Differential Equations I

for Students of Engineering Sciences

Homework 3

Problem 1: Write the following equations as *similarity differential equations* and solve them by a suitable change of variables. Are the solutions defined for all $t > 0$?

(a)

$$tu(t)^2u'(t) = u(t)^3 + t^3, \quad t > 0.$$

(b)

$$tu'(t) - u(t) = te^{u(t)/t}, \quad t > 0.$$

Problem 2: Consider the following *Riccati differential equation*:

$$u'(t) = \frac{1}{t-t^2}u(t) - \frac{t}{t-t^2}u(t)^2 + \frac{t-1}{t-t^2}, \quad t \in (0, 1).$$

(a) Find *one particular* solution u_0 for this equation.

Hint: There exists a constant solution.

(b) Find the general solution of the equation.