Differential Equations I for Students of Engineering Sciences Work sheet 6

Problem 1:

(a) Determine the equilibria of the following systems of differential equations. Check whether the equilibria are, unstable, stable, or asymptotically stable.

(i)
$$\binom{u_1}{u_2}' = \binom{1}{1} \frac{5}{u_2} \binom{u_1}{u_2} + \binom{7}{-9},$$
 (ii) $\binom{u_1}{u_2}' = \binom{1}{-1} \frac{1}{1} \binom{u_1}{u_2}$
(iii) $\binom{u_1}{u_2}' = \binom{-1}{5} \frac{-2}{1} \binom{u_1}{u_2} - \binom{6}{6},$ (iv) $\binom{u_1}{u_2}' = \binom{-1}{-1} \frac{0}{u_2} \binom{u_1}{u_2}.$

(b) For which $\alpha \in \mathbb{R}$ is $(0,0,0)^{\top} \in \mathbb{R}^3$ a stable equilibrium of u' = Au? Here, A is respectively given by

(i)
$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & \alpha & -1 \\ 0 & 1 & \alpha \end{pmatrix}$$
, (ii) $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & \alpha & -1 \\ 0 & 1 & \alpha \end{pmatrix}$, (iii) $A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & \alpha & 1 \\ 0 & 1 & \alpha \end{pmatrix}$.

Problem 2:

(a) (Problem from an old exam, 4 points) Consider the initial value problem

$$u''(t) + 4u'(t) + 3u(t) = 2\cos(t) + t^2 e^{-2t} \quad \text{for } t > 0,$$

with

$$u(0) = 0, \qquad u'(0) = 5.$$

What algebraic equation do you get when you apply the Laplace transform to this problem?

(b) Let $F(s) = \frac{1}{s(s+1)^2}$ be the image of the function $f: (0,\infty) \to \mathbb{R}, t \mapsto f(t)$ under the Laplace transform. Determine f.