Fachbereich Mathematik der Universität Hamburg **WiSe 2024/25**
Prof. Dr. T. Schmidt, Dr. C. Goetz

Differential Equations I for Students of Engineering Sciences

Work sheet 5

Problem 1. Consider the matrix $A \in \mathbb{R}^{3\times 3}$,

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 4 & -3 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

(a) Compute the eigenvalues of A. Determine the corresponding eigenvectors, and, if necessary, the generalized eigenvectors.

(b) Determine a fundamental system for $u' = Au$.

Problem 2.

(a) We consider the inhomogeneous system

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}' = \begin{pmatrix} 3 & 4 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 10t\mathrm{e}^{-3t} \end{pmatrix}.$$

Find a fundamental system for the homogeneous problem. Find a particular solution of the inhomogeneous problem by the method of variation of constants.

(b) We consider the inhomogeneous system

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}' = \begin{pmatrix} -6 & -4 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} + \mathrm{e}^{2t} \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

Determine a *real* fundamental system for the homogeneous problem.Find a particular solution of the inhomogeneous problem by using the ansatz
$u_p(t) = \mathrm{e}^{2t}(a, b)^\top$, with suitable $a, b \in \mathbb{R}$.

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