Differential Equations I
for Students of Engineering Sciences
Work sheet 4

Problem 1: Find the general solutions of the following differential equations:

(a) $u'''(t) - u'(t) = 0$;

(b) $u'''(t) - 5u''(t) + 8u'(t) - 4u(t) = 0$;

(c) $u''(t) - 2u'(t) + 5u(t) = 0$.

Problem 2 (*problem from an old exam, 4 points*): Consider the third order differential equation
$$u'''(t) + a_2 u''(t) + a_1 u'(t) + a_0 u(t) = 0 \qquad (*)$$
with real coefficients $a_0, a_1, a_2 \in \mathbb{R}$. Check whether the following sets of functions can be fundamental systems for the solution space of this equation (with suitable coefficients $a_0, a_1, a_2 \in \mathbb{R}$). Give an explanation for your answers.

(a) $M_1 := \{u_1(t) = -t,\ u_2(t) = 1,\ u_3(t) = 2t\}$.

(b) $M_2 := \{u_1(t) = \mathrm{e}^{-t},\ u_2(t) = \mathrm{e}^t,\ u_3(t) = \mathrm{e}^{2t},\ u_4(t) = \mathrm{e}^{3t}\}$.

(c) $M_3 := \{u_1(t) = \mathrm{e}^{-t},\ u_2(t) = e^{\mathrm{i}t},\ u_3(t) = e^{2\mathrm{i}t}\}$.

(d) $M_4 := \{u_1(t) = 1,\ u_2(t) = e^{-2\mathrm{i}t},\ u_3(t) = e^{2\mathrm{i}t}\}$

Problem 3: Solve the initial value problem
$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}, \qquad \begin{pmatrix} u_1(0) \\ u_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

Dates of classes: 09. December 2024 - 13. December 2024