Fachbereich Mathematik der Universität Hamburg **WiSe 2024/25**
Prof. Dr. T. Schmidt, Dr. C. Goetz

Differential Equations I for Students of Engineering Sciences

Work Sheet 3

Problem 1:

(a) Consider the equation

$$y''(t) - 8y'(t) + 15y(t) = 0, \qquad t > 0.$$

Find $\lambda_1, \lambda_2 \in \mathbb{R}$, such that $y_1(t) := c_1 e^{\lambda_1 t}$ and $y_2(t) := c_2 e^{\lambda_2 t}$, with $c_1, c_2 \in \mathbb{R}$, are solutions of this equation. Is $y_1 + y_2$ also a solution?

(b) Now let the following *Euler differential equation* be given:

$$t^2 u''(t) - 7tu'(t) + 15u(t) = 0, \qquad t > 0,$$

Solve this equation by using a suitable change of variables to transform it to the equation from part (a).

Problem 2: (*problem from an old exam, 5 points*)

(a) Check whether the following ordinary differential equations are exact.

(i) $y(t)^2 + (t^2 y(t) - 1)y'(t) = 0$;

(ii) $2ty(t)^2 + (2y(t) + 2t^2 y(t))y'(t) = 0$.

(b) Determine a corresponding scalar potential and the general solution for the exact equation from part (a).

Problem 3: Show that the differential equation

$$(t^2 - 1)y + (t^3 + t)y' = 0, \qquad t > 0,$$

admits an integrating factor h that depends only on t (i.e. $h = h(t)$) and determine the solutions of the equation.

Dates of classes: 25. November 2024 - 29. November 2024