Fachbereich Mathematik der Universität Hamburg **WiSe 2024/25**
Prof. Dr. T. Schmidt, Dr. C. Goetz

Differential Equations I for Students of Engineering Sciences

Exercise class 1

Exercise 1:

(a) We consider the differential equation

$$u'(t) = u(t) - t \qquad \text{for } t \in \mathbb{R}.$$

Which of the following functions defined on $\mathbb{R}$ are solutions of such differential equation ?

$$\begin{array}{ll} (1)\ u_1(t) := \mathrm{e}^t, & (2)\ u_2(t) := t + 1, \\ (3)\ u_3(t) := \alpha \mathrm{e}^t + t + 1, & (4)\ u_4(t) := \mathrm{e}^t + \beta(t+1), \end{array}$$

where $\alpha, \beta \in \mathbb{R}$ are arbitrary constants.

(b) Given the differential equation

$$y''(t) + 16y(t) = 0 \qquad \text{for } t \in \mathbb{R},$$

show that the functions

$$y_1(t) := \cos(4t), \qquad y_2(t) := \sin(4t), \qquad y_3(t) := \alpha\cos(4t) + \beta\sin(4t)$$

defined on $\mathbb{R}$ for arbitrary $\alpha, \beta \in \mathbb{R}$, are solutions.

Does the initial value problem $y'' + 16y = 0$, $y(0) = 0$ have a unique solution?

Exercise 2: We denote the humidity of a cloth in a drying machine at time $t \geq 0$ with $m(t)$, and let $m_0 := m(0) > 0$. It holds:

- During drying, the decrease in humidity is proportional to the humidity.
- After 15 minutes the cloth has still $50\,\%$ of its original humidity m_0.

Set an appropriate differential equation describing this process and determine its solution. How long does it take until the cloth has just the $2\,\%$ of its original humidity?

Exercise 3: We consider the function $f : [0, \infty) \longrightarrow \mathbb{R}$, defined by

$$f(u) := (1-u)u.$$

(a) Show that the function φ_c defined by

$$\varphi_c(t) := \frac{e^t}{e^t + c},$$

with $c \in \mathbb{R}$ is a solution of the differential equation $\varphi_c' = f(\varphi_c)$ for $t \geq 0$, provided $e^t + c > 0$.

(b) Assume now that an initial value $u(0) = u_0 > 0$ is given. How should c from Part (a) be chosen, such that $u(t) = \varphi_c(t)$ solves the initial value problem $u' = f(u)$, $u(0) = u_0$? What are the solutions for $u_0 = 0$ and for $u_0 = 1$?

(c) Are the solutions in (b) with $u_0 > 0$ defined for every $t > 0$? Meaning, is the condition $e^t + c > 0$ satisfied for all $t > 0$?

(d) Determine the solutions of the initial value problem $u' = f(u)$, $u(0) = u_0$ for $u_0 = \frac{1}{2}$ and for $u_0 = \frac{3}{2}$. Sketch these solutions.

How do the solutions behave for $t \to \infty$?

Dates of classes: 28. October 2024 - 01. November 2024