

Mathematics III Exam
(Module: Differential Equations I)
March 4, 2025

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

Surname:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

First name:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Matr.-No.:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

BP:

CI	CS	DS	ES	GES	
----	----	----	----	-----	--

Assessment according to examin. reg:

with Analysis III

single scoring

I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam’s beginning in retrospect.

(Signature)

Problem	Points	Evaluator
1		
2		
3		
4		

$\sum$ =

Problem 1. (5 points)

(a) Solve the following initial value problem:

$$u'(t) = -(2t + 1)u(t) + e^{-t^2} \quad \text{for } t \geq 0, \quad u(0) = 1.$$

(b) Solve the following initial value problem by applying a suitable substitution:

$$u'(t) = (2t + 1)u(t) - e^{-t^2}(u(t))^2 \quad \text{for } t \geq 0, \quad u(0) = 1.$$

Problem 2. (5 points)

Consider the differential equation

$$u'''(t) + 2u''(t) = 0 \quad \text{for } t \geq 0. \quad (*)$$

- (a) Determine the general solution of this equation.
- (b) Is the initial condition $u(0) = 1$ enough to characterize a unique solution of $(*)$? Explain your answer.
- (c) For which $a, b \in \mathbb{R}$ does the solution u of the initial value problem for $(*)$ with

$$u(0) = 1, \quad u'(0) = a, \quad u''(0) = b$$

satisfy the condition

$$\lim_{t \rightarrow \infty} u(t) = 0?$$

Problem 3. (4 points)

Consider the differential equation

$$(t - \sin(t) \cos(t))u^2 + (t^2 + \cos^2(t) + 1)u \cdot u' = 0 \quad \text{for } t \geq 0.$$

- (a) Show that this is an exact differential equation.
- (b) Determine a potential for this equation.
- (c) Solve the corresponding initial value problem with $u(0) = 1$.

Problem 4. (6 points)

Consider the system of differential equations $u' = Au$ with

$$A = \begin{pmatrix} -3 & 0 & 0 \\ 1 & -1 & 2 \\ 2 & 0 & 1 \end{pmatrix}.$$

- (a) Determine a fundamental system for this equation.
- (b) Determine all equilibria of the system and check them for stability.

