Mathematics Department
Prof. Dr. T. Schmidt

SoSe 2025

Mathematics III Exam
(Module: Differential Equations I)
August 27, 2025

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

Surname:																
First name:																
Matr.-No.:																

BP:	CI	CS	DS	ES	GES	

Assessment according to examin. reg: with Analysis III ☐ single scoring ☐

I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

(Signature)

Exercise	Points	Evaluator
1		
2		
3		
4		

$\sum$ =

Problem 1. (5 points)

For each of the following differential equations, find the general solution by using the method of separation of variables.

After that, solve the corresponding initial value problems with the given initial data and determine for which t the respective solutions exist.

(a) $2t^2u' = u^2$ for $t \geq 1$, $u(1) = 4$,

(b) $u'u = \mathrm{e}^t$ for $t \geq 0$, $u(0) = 2$.

Problem 2. (6 points)

Consider the matrix

$$A = \begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}.$$

(a) Determine a fundamental system for the solution of $u' = Au$.

(b) Determine all equilibria of $u' = Au$ and check them for stability.

Problem 3. (4 points)

Let $A \in \mathbb{R}^{4\times 4}$ be a real matrix with eigenvalues

$$\lambda_1 = -2, \quad \lambda_2 = -1, \quad \lambda_3 = 2\mathrm{i}, \quad \lambda_4 = -2\mathrm{i}.$$

Which of the following statements are true, which are false? Explain your answers.

(1) The system $u' = Au$ has exactly one equilibrium.

(2) The equilibrium $u^* = (0,0,0,0)^\top \in \mathbb{R}^4$ is asymptotically stable.

(3) There exists a π-periodic solution of $u' = Au$.

(4) There exists a solution u of $u' = Au$ with $\lim_{t\to\infty} |u(t)| = \infty$.

Hint: In each case a short explanation is enough. You do not need long calculations.

Problem 4. (5 points)

Consider the differential equation

$$u''(t) - 2u'(t) + 2u(t) \;=\; 2\sin(t) - 4\cos(t).$$

(a) Determine a *real* fundamental system for the homogenous problem.

(b) Find a particular solution u_p of the inhomogeneous problem.

Hint: You can use the ansatz $u_p(t) = a\cos(t) + b\sin(t)$ with $a, b \in \mathbb{R}$.