Fachbereich Mathematik der Universität Hamburg **WiSe 2023/24**
Prof. Dr. T. Schmidt,
Dr. H. P. Kiani, E. Ficola

Differential Equations I for Students of Engineering Sciences

Sheet 7, Homework

Exercise 1: Consider the initial value problem be given.

$$u''(t) - u'(t) - 2u(t) = e^{2t} \cdot \sin(t), \qquad u(0) = u'(0) = 0.$$

a) Solve the initial value problem with the help of the Laplace transformation.

b) Determine the solution of the initial value problem without Laplace transformation. Proceed as follows.

(i) Determine the general solution of the corresponding homogeneous differential equation with help of the characteristic polynomial.

(ii) Rewrite the differential equation as a first order system and provide a fundamental matrix for this system.

(iii) Determine the general solution of the inhomogeneous differential equation. Use the method of variation of constants for the corresponding system.

(iv) Adjust the coefficients to the initial conditions.

Hint:

- $\displaystyle\int e^{\alpha t} \cdot \sin(t)\, dt = \frac{e^{\alpha t}}{\alpha^2 + 1} \left(\alpha \cdot \sin(t) - \cos(t)\right) + C\,.$
- Use Part b) as recap and a combination of techniques from the previous sheets!

Exercise 2:

Consider the linear system $\boldsymbol{u}\,'(t) = \begin{pmatrix} -2 & 1 & -2\alpha \\ 0 & -1+\alpha & 0 \\ -2\alpha & -1 & -2 \end{pmatrix} \boldsymbol{u}\,(t)\,.$

Analyze the stability behavior of the stationary point $(0,0,0)^T$ depending on the parameter $\alpha \in \mathbb{R}\,.$

Exercise 3:

In a two-population model (predator-prey model) $x(t)$ denotes the population of the prey species, $y(t)$ the population of the predator species at time t. The temporal growth of the populations is described by the following system of differential equations

$$\begin{aligned} x' &= x\,(x - 1 - y) \\ y' &= y\,(2x - 3 - y). \end{aligned}$$

a) Find all equilibrium points of the system.

b) Analyze the stability of every equilibrium point.

Hand in until: 26.01.2024