Fachbereich Mathematik der Universität Hamburg **WiSe 2023/24**
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Differential Equations I for Students of Engineering Sciences

Sheet 6, Homework

Exercise 1:

Consider the system of differential equations

$$\boldsymbol{u}'(t) = \begin{pmatrix} -3 & 2 \\ -8 & -3 \end{pmatrix} \boldsymbol{u}(t) + \begin{pmatrix} 20 \\ 20 \end{pmatrix}.$$

a) Determine a real fundamental system of the corresponding homogeneous system of differential equations .

b) With the help of an appropriate ansatz determine a particular solution of the inhomogeneous system and provide the general solution of the inhomogeneous differential equation.

Exercise 2)

Consider the system of differential equations

$$\boldsymbol{u}'(t) = \boldsymbol{A} \cdot \boldsymbol{u}(t) = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & -2 \end{pmatrix} \cdot \boldsymbol{u}(t).$$

a) Determine the general solution of the system.

b) Determine the solution $\boldsymbol{u}(t)$ of the corresponding initial value problem with

$$\boldsymbol{u}(0) = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$$

and compute for this solution $\lim\limits_{t\to\infty} \boldsymbol{u}(t)$.

c) Does the solution of the system from part a) converge to zero for $t \to \infty$ for every initial conditions? Justify your answer.

Exercise 3:

Consider the linear differential equation of second order

$$u''(t) + \frac{7}{t}u'(t) + \frac{9}{t^2}u(t) = 0.$$

a) With the help of the ansatz: $u_0(t) = t^k$, determine a solution of the differential equation.

b) With the help of te reduction ansatz $\hat{u}(t) = u_0(t)\cdot w(t)$ find another solution of the differential equation and provide the general solution of the differential equation.

c) Compute the solution of the boundary problem

$$u''(t) + \frac{7}{t}u'(t) + \frac{9}{t^2}u(t) = 0, \qquad 1 < t < e^{\frac{1}{3}}, \qquad u(1) = 0,\ u(e^{\frac{1}{3}}) = 1.$$

d) Can you also calculate a solution of the following boundary value problem?

$$u''(t) + \frac{7}{t}u'(t) + \frac{9}{t^2}u(t) = 0, \qquad 1 < t < e^{\frac{1}{3}}, \qquad u(1) = 0,\ u'(e^{\frac{1}{3}}) = 1.$$

Hand in until: 12.01.2024