

Differential Equations I
for Students of Engineering Sciences
Sheet 6, Exercise class

Exercise 1: For each of the following matrices

$$\mathbf{A}^{[1]} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}, \quad \mathbf{A}^{[2]} = \begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}, \quad \mathbf{A}^{[3]} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \quad \mathbf{A}^{[4]} = \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix},$$

determine a real fundamental system of the solution space of

$$\mathbf{y}'(t) = \mathbf{A}^{[k]} \mathbf{y}(t), \quad k = 1, 2, 3, 4.$$

Exercise 2) Consider the system of differential equations

$$\mathbf{u}' = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix} \mathbf{u} + \begin{pmatrix} 4t \\ t \end{pmatrix} \quad t \geq 0.5.$$

a) Show that

$$\mathbf{U}(t) := \begin{pmatrix} t^{-2} & t \\ -2t^{-2} & t \end{pmatrix}$$

is a fundamental system of the corresponding homogeneous system of differential equations .

b) Determine the general solution of the inhomogeneous problem.

c) Determine the solution of the corresponding initial value problem with initial values $\mathbf{u}(1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Dates of classes: 08.01-12.01.2024