Fachbereich Mathematik der Universität Hamburg **WiSe 2023/24**
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Differential Equations I
for Students of Engineering Sciences

Sheet 4, Homework

Exercise 1:

a) Which of the following differential equations for $u(t)$ are exact?

(i) $u + u' = 0\,.$

(ii) $(12tu + 3) + 6t^2 \cdot u' = 0\,.$

(iii) $2t(u^2 - t^2 - 1) + 2uu' = 0\,.$

(iv) $u^3 + e^t + 3tu^2u' = 0\,.$

b) For the exact differential equations in Part a) determine a corresponding potential and the general solution.

Exercise 2:

a) Determine the solution to the initial value problem

$$u''(t) + 2t^3\,u'(t) = e^{-\frac{t^4}{2}} \cdot \sin(2t) \qquad u(0) = 2,\ u'(0) = 0.$$

Hint: It is sufficient to specify an integral representation of the solution.

b) Solve the initial value problem

$$u''(t) = (u(t))^{-3} = g(u(t)), \qquad u(0) = 2,\ u'(0) = 0.$$

Exercise 3:

The speed at which a solid substance dissolves in a solvent is proportional to the still undissolved quantity of the substance $S(t)$ at time t and to the difference between the saturation concentration and the actual concentration of the already dissolved substance. Let

$V :=$ volume $\quad K_M :=$ saturation concentration,

$K_0 :=$ initial concentration $\quad S(t) :=$ undissolved quantity of the substance S at time t,

$S_0 := S(0) =$ undissolved quantity of the substance S at time zero (initial value),

$K_0 + \dfrac{S_0 - S(t)}{V} =$ concentration of S at time t,

$\gamma :=$ proportionality constant.

a) Describe the diffusion process through a differential equation for $S(t)$.

b) Determine the solution of the initial value problem with data
$S_0 = 10$ kg, $V = 100$ lit, $K_M = 0.25\,\text{kg/lit}$, $K_0 = 0\,\text{kg/lit}$, $\gamma = 4\,\text{lit}/(\text{kg}\cdot\text{s})$.
Use the substitution known from the lecture for logistic growth $u = S^{-1}$.

Hand in until: 01.12.2023