

Differential Equations I for Students of Engineering Sciences Sheet 3, Homework

Exercise 1:

- a) Determine the solution of the following initial value problem

$$y'(t) = t + t(y(t))^2 \quad \text{for } t > 0, \quad y(0) = 1.$$

On which interval $I = [0, t^*)$ is its solution defined?

- b) Find the general solution of the following differential equation.

$$y'(t) = e^{-2t} \cdot \sqrt[3]{y(t)}.$$

- c) Which solution results in b) if the initial value $y(0) = 1$ is given?
d) Which solution results in b) if the initial value $y(0) = 0$ is given?

Exercise 2:

- a) Consider the differential equation

$$y'(t) = f(\alpha t + \beta y(t) + \gamma)$$

with $\alpha, \beta, \gamma \in \mathbb{R}$ and $\alpha + \beta f(\alpha t + \beta y(t) + \gamma) \neq 0$.

Show that with the help of the substitution

$$u(t) := \alpha t + \beta y(t) + \gamma$$

it can be transformed into a separable differential.

- b) Determine the general solution of the differential equation

$$y' = 1 + \frac{2}{t - y + 4}, \quad \text{for } t - y + 4 > 0.$$

- c) Check the solution obtained in Part b) by inserting it into the differential equation.

Hand in until: 17.11.2023