Fachbereich Mathematik der Universität Hamburg **WiSe 2023/24**
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Differential Equations I for Students of Engineering Sciences

Sheet 2, Homework

Exercise 1:

We consider the differential equation $y' - 2y = 4t$.

a) Show that $y_p(t) = -2t - 1$ solves the differential equation.

b) Let $\tilde{y}$ be a second solution of the differential equation.. Show that $y := \tilde{y} - y_p$ solves the corresponding homogeneous differential equation $y' - 2y = 0$.

c) Compute solutions y_h of the corresponding homogeneous differential equation $y_h' - 2y_h = 0$, for example with the help of the formula on page 9 of the lecture.

d) From b) it follows that every solution of the inhomogeneous differential equation can be written as sum of a solution of the homogeneous differential equation and y_p. Calculate the solution of the initial value problem

$$y' - 2y = 4t, \qquad y(0) = 2.$$

Exercise 2:

Analogously to the procedure on page 9 of the lecture, prove the representation given on page 11

$$u(t) = e^{A(t)}(B^*(t) + C), \qquad \text{with } A'(t) = a(t),\ (B^*(t))' = e^{-A(t)} \cdot b(t),\ C \in \mathbb{R} \qquad (*)$$

for any solution of the differential equation

$$u'(t) = a(t) \cdot u(t) + b(t).$$

Where the same assumptions of the lecture apply to a and b.

Hint:
Prove first, by a substitution into the differential equation , that a solution is given by $(*)$.

Conversely, assume that you have a solution u and, analogous to the procedure on page 9 of the lecture, show that $e^{-A(t)}u(t) - B^*(t)$ is constant.

Exercise 3:

Determine a solution of the initial value problem

$$u'(t) = \cos(t) \cdot u(t) + te^{\sin(t)}, \qquad u(0) = 5.$$

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