

Mathematics III Exam
(Module: Differential Equations I)

March 4, 2024

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

Surname:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

First name:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Matr.-No.:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

BP:

AIW	BU	BV	CI CS	ET	EUT	GES	IN IIW	LUM	MB	MTB MEC	SB	VT	
-----	----	----	----------	----	-----	-----	-----------	-----	----	------------	----	----	--

Assessment according to examin. reg:

with Analysis III	
-------------------	--

single scoring	
----------------	--

I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

(Signature)

Exercise	Points	Evaluator
1		
2		
3		
4		

$\Sigma =$

Exercise 1: (5 points)

Compute the general solution of the following differential equation

$$u'(t) = \sin(2t)u(t) + e^{\cos(2t)}(u(t))^3.$$

Hint: It is useful to employ a standard substitution.

Exercise 2: (4 points)

Consider the initial value problem

$$u'''(t) - 5u''(t) + 2u(t) = 3 + \cos(t), \quad u(0) = 4, u'(0) = 3, u''(0) = 0.$$

- a) What is the order of the differential equation?
- b) Is it an explicit equation? If this is not the case, provide an equivalent explicit differential equation.
- c) Reformulate the initial value problem as an equivalent initial value problem for a system of first order.

Exercise 3: (4 points)

Consider the following differential equation of order three

$$u'''(t) + a_2u''(t) + a_1u'(t) + a_0u(t) = 0 \quad (*)$$

with real coefficients $a_0, a_1, a_2 \in \mathbb{R}$. Examine for each of the following sets of functions, if it might be (for suitable coefficients $a_0, a_1, a_2 \in \mathbb{R}$) a fundamental system for the solution space of the differential equation.

Justify your answers.

- a) $M_1 := \{u_1(t) = -t, u_2(t) = 1, u_3(t) = 2t\}$.
- b) $M_2 := \{u_1(t) = e^{-t}, u_2(t) = e^t, u_3(t) = e^{2t}, u_4(t) = e^{3t}\}$.
- c) $M_3 := \{u_1(t) = e^{-t}, u_2(t) = e^{it}, u_3(t) = e^{2it}\}$.
- d) $M_4 := \{u_1(t) = 1, u_2(t) = e^{-2it}, u_3(t) = e^{2it}\}$.

Exercise 4 (7 points)

Consider the system of differential equations

$$\mathbf{u}'(t) = \begin{pmatrix} 3 & -5 \\ 5 & -5 \end{pmatrix} \mathbf{u}(t).$$

- a) Analyse the stability of the stationary point $(0, 0)^T$ of the system.
- b) Determine a real-valued fundamental system of the system of differential equations.

