Fachbereich Mathematik der Universität Hamburg **Winter Term 2022/23**
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Differential Equations I for Students of Engineering Sciences

Sheet 7, Exercise class

Exercise 1: Consider the boundary value problem

$$y'' + 2y' + y = h(x) \qquad x \in]0,1[$$

$$y(0) - y(1) = \gamma_1$$

$$\alpha y'(0) + 2y(1) = \gamma_2 \qquad \alpha, \gamma_1, \gamma_2 \in \mathbb{R}\,.$$

For which values of α is the boundary problem uniquely solvable for arbitrary $\gamma_1, \gamma_2 \in \mathbb{R}$ and arbitrary functions $h(x)$ continuous on the interval $[0,1]$?

Exercise 2:

a) Analyze the stability of the stationary point $(0,0)^T$ of the linear system $\dot{\boldsymbol{y}} = A\,\boldsymbol{y}$.

i) $A = \begin{pmatrix} 5 & -2 \\ 6 & -3 \end{pmatrix}$, **ii)** $A = \begin{pmatrix} -2 & 0 \\ 4 & -2 \end{pmatrix}$,

iii) $A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$, **iv)** $A = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$.

b) For which of the following matrices $\boldsymbol{A}$ can you exclude a stable stationary point (equilibrium point) of the system of differential equations $\dot{\boldsymbol{y}}(t) = \boldsymbol{A}\,\boldsymbol{y}(t)$ without knowing the value of $\gamma \in \mathbb{R}$?

i) $\boldsymbol{A} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & \gamma & -1 \\ 0 & 1 & \gamma \end{pmatrix}$, **ii)** $\boldsymbol{A} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & \gamma & -1 \\ 0 & 1 & \gamma \end{pmatrix}$, **iii)** $\boldsymbol{A} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & \gamma & 1 \\ 0 & 1 & \gamma \end{pmatrix}$.

Exercise 3:

The Van-der-Pol equation

$$\ddot{x} - \epsilon\,(1 - x^2)\,\dot{x} + x = 0 \qquad \epsilon \in \mathbb{R}^+$$

describes the behavior of a Van-der-Pol oscillator. It is an oscillator with non-linear damping and self-excitation. For small displacements x the damping is negative, and for large displacements x the damping is positive. There is no exact analytic solution. Analyze the stability of the equilibrium point $x = 0$.

Hint: Rewrite the differential equation as an equivalent system.

Dates of classes: 23.01.-27.01.2023