Differential Equations I
for Students of Engineering Sciences

Sheet 7, Homework

Exercise 1:

Consider the boundary value problem

$$y''(x) + 2\,y'(x) + 2y(x) = h(x) \qquad t \in\,]0, \pi[$$

$$y(0) + \alpha\,y(\pi) = r_1$$

$$y'(0) = r_2 \qquad \alpha,\, r_1,\, r_2 \in \mathbb{R}\,.$$

a) Determine a real representation for the general solution of the homogeneous differential equation
$$y''(x) + 2\,y'(x) + 2y(x) = 0.$$

b) For which values of α is the boundary problem uniquely solvable for arbitrary values $r_1,\, r_2 \in \mathbb{R}$ and arbitrary functions $h(x)$ continuous on the interval $[0, \pi]$?

c) Determine the solution of the boundary problem for

$$h(x) = 3 + x,\, \alpha = -1,\, r_1 = -\frac{\pi}{2},\, r_2 = 1\,.$$

Exercise 2:

Consider the linear system $\boldsymbol{y}'(t) = \begin{pmatrix} -2 & 1 & -2\alpha \\ 0 & -1+\alpha & 0 \\ -2\alpha & -1 & -2 \end{pmatrix} \boldsymbol{y}(t)\,.$

Analyze the stability of the stationary point $(0,0,0)^T$ depending on the parameter $\alpha \in \mathbb{R}\,.$

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