Fachbereich Mathematik der Universität Hamburg **Winter Term 2022/23**
Prof. Dr. J. Behrens, Dr. H. P. Kiani,
E. Ficola

Differential Equations I
for Students of Engineering Sciences

Sheet 5, Homework

Exercise 1:

Consider the Euler differential equation

$$x^2 y''(x) \, - \, 3xy'(x) \, + 4y(x) \, = \, 0 \, , \qquad x > 0.$$

a) Does the ansatz $u(x) = x^r$ from Sheet 3 lead to a fundamental system?

b) Let u be a solution of Part a). With the help of the reduction ansatz $\tilde{y}(x) = u(x) \cdot z(x)$ determine another solution of the differential equation and provide its general solution.

Exercise 2: Consider the differential equation

$$y'''(t) - 4y'(t) = \, e^{2t} \cdot \sin(t) \, + \, e^{-2t} \cdot \sin(t) \, .$$

a) Determine the general solution of the corresponding homogeneous differential equation.

b) Rewrite the differential equation as a system of first order equations and provide a fundamental matrix for this system.

c) Determine the general solution of the inhomogeneous differential equation. Apply the variation of constants method to the corresponding system.

Hint: $\int e^{\alpha t} \cdot \sin(t) \, dt \, = \, \dfrac{e^{\alpha t}}{\alpha^2 + 1} \, (\alpha \cdot \sin(t) - \, \cos(t)) \, + \, C \, .$

Hand in until: 16.12.2022