Fachbereich Mathematik der Universität Hamburg **Winter Term 2022/23**
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Differential Equations I
for Students of Engineering Sciences

Sheet 4, Exercise class

Exercise 1: With the matrices

$$\boldsymbol{A}^{[1]} = \begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}, \qquad \boldsymbol{A}^{[2]} = \begin{pmatrix} 5 & \frac{1}{2} \\ 0 & 5 \end{pmatrix}, \qquad \boldsymbol{A}^{[3]} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}, \qquad \boldsymbol{A}^{[4]} = \begin{pmatrix} 5 & 2 \\ -2 & 5 \end{pmatrix},$$

compute real fundamental systems of the solution spaces of

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{A}^{[k]}\, \boldsymbol{y}(t), \qquad k = 1, 2, 3, 4.$$

Exercise 2: Rewrite each of the following initial value problems as an initial value problem for a first order system.

a) $x^2 y''(x) - 3xy'(x) - 4y(x) = 0, \qquad y(1) = 4,\ y'(1) = 4.$

b) $\frac{d^3}{dt^3} y = 2y - \dot{y} + 2\ddot{y} + 3, \qquad y(0) = 7,\ \dot{y}(0) = 0,\ \ddot{y}(0) = 5.$

Exercise 3: In this exercise a little computation might be necessary in part b). Parts a) and c) can be answered without any calculation!

a) Do the functions

$$x_1(t) = e^t, \qquad x_2(t) = te^t, \qquad x_3(t) = e^{2t}, \qquad x_4(t) = e^{3t}$$

define a fundamental system for the space of solutions of the differential equation

$$\frac{d^3}{dt^3} x(t) - 6\ddot{x}(t) + 11\dot{x}(t) - 6x(t) = 0\,?$$

b) The functions

$$\boldsymbol{x}^{[1]}(t) = \begin{pmatrix} t^2 - 2t \\ 2(t-1) \\ 2 \end{pmatrix}, \qquad \boldsymbol{x}^{[2]}(t) = \begin{pmatrix} 1 - t \\ -1 \\ 0 \end{pmatrix}, \qquad \boldsymbol{x}^{[3]}(t) = \begin{pmatrix} t^2 - 2 \\ 2t \\ 2 \end{pmatrix}$$

are solutions of the system

$$\dot{\boldsymbol{x}}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \boldsymbol{x}(t).$$

Do they also constitude a fundamental system?

c) Do the functions

$$x_1(t) = e^t, \qquad x_2(t) = e^{2t}, \qquad x_3(t) = e^{it}$$

define a fundamental system for the space of solutions of the differential equation

$$\frac{d^3}{dt^3}x(t) + a_2\ddot{x}(t) + a_1\dot{x}(t) + a_0x(t) = 0 \qquad \text{with real coefficients } a_0, a_1, a_2 \in \mathbb{R} \text{ ?}$$

Justify your answers!

Dates of classes: 28.11.-02.12.2022