Fachbereich Mathematik der Universität Hamburg **Winter Term 2022/23**
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Differential Equations I
for Students of Engineering Sciences

Sheet 4, Homework

Exercise 1:

Consider the system of differential equations

$$\dot{\boldsymbol{x}}(t) = \begin{pmatrix} 0 & 1 \\ \frac{3}{t^2} & \frac{1}{t} \end{pmatrix} \boldsymbol{x}(t) + \begin{pmatrix} t \\ 3 \end{pmatrix}, \qquad t \geq 0.5.$$

The functions

$$\boldsymbol{x}^{[1]}(t) = \begin{pmatrix} -\frac{1}{t} \\ \frac{1}{t^2} \end{pmatrix} \text{ and } \boldsymbol{x}^{[2]}(t) = \begin{pmatrix} t^3 \\ 3t^2 \end{pmatrix}$$

are solutions of the associated homogeneous differential system.

a) Do $\boldsymbol{x}^{[1]}$ and $\boldsymbol{x}^{[2]}$ form a fundamental system of the associated homogeneous differential equation ?

b) Determine a particular solution of the inhomogeneous differential equation by the variation of constants method and provide the general solution of the inhomogeneous differential equation.

c) Compute the solution of the corresponding initial value problem with initial values $\boldsymbol{x}(1) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

Exercise 2: (7 + 3 points)

Let $\boldsymbol{A} = \begin{pmatrix} -1 & 0 & 4 \\ 0 & -5 & 0 \\ -4 & 0 & -1 \end{pmatrix}$.

a) Determine a real representation of the general solution of the homogeneous system of differential equations

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A} \cdot \boldsymbol{x}(t).$$

b) With the help of a suitable ansatz, determine the general solution of the inhomogeneous system of differential equations

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A} \cdot \boldsymbol{x}(t) + e^{3t} \cdot \begin{pmatrix} 4 \\ 8 \\ -4 \end{pmatrix}.$$

Hand in until: 02.12.2022