Fachbereich Mathematik der Universität Hamburg **Winter Term 2022/23**
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Differential Equations I for Students of Engineering Sciences

Sheet 3, Exercise class

Exercise 1:

Consider the homogeneous Euler differential equation

$$x^2 y''(x) + a_1\, x\, y'(x) + a_0\, y(x) = 0, \qquad (*)$$

with real coefficients a_k and $x > 0$.

a) Let y_- and y_+ be two different solutions of the differential equation $(*)$. Show that every real or complex linear combination of y_- and y_+ solves the differential equation $(*)$ as well.

b) Using the Ansatz $y(x) = x^r$ determine two real linearly independent solutions for each of the following second order differential equations.

i) $x^2 y'' - 3xy' + 3y = 0$, **ii)** $x^2 y'' - 3xy' + 5y = 0$.

If you obtain complex solutions, note that due to part a) the real and imaginary parts of the complex solution are real solutions of the differential equations.

Exercise 2:

a) Using the substitution $u(t) := \dot{y}(t)$ and subsequent separation of variables determine the general solution of the following second order differential equation.

$$\ddot{y}(t)\,\sin(t) = \cos(t)\dot{y}(t) + \cos(t)\,.$$

Insert it into the differential equation, to verify that you have actually found a solution for every $t \in \mathbb{R}$.

b) Consider the differential equation from Part a).

(i) Determine the solution of the corresponding initial value problem with initial values

$$y(\frac{\pi}{6}) = \frac{\pi}{3}\,, \qquad \dot{y}(\frac{\pi}{6}) = -1\,.$$

(ii) Now consider the initial values

$$y(0) = 0\,, \qquad \dot{y}(0) = 0\,,$$

and

$$y(0) = 0\,, \qquad \dot{y}(0) = -1\,,$$

respectively. Can you determine unique solutions here as well?
Are your results consistent with the existence and uniqueness theorem from the lecture?

Dates: 14.11.-18.11.2022