Fachbereich Mathematik der Universität Hamburg **Winter Term 2022/23**
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Differential Equations I for Students of Engineering Sciences

Sheet 3, Homework

Exercise 1:

a) Determine a solution $y : [0,1[\to \mathbb{R}$ of the initial value problem

$$\ddot{y}(t) \;=\; \dot{y}(t)\,(y(t)+1), \qquad y(0)=1, \quad \dot{y}(0)=2\,\cdot$$

b) Determine all radial symmetric solutions $u = u(r)$, $r \;=\; \sqrt{x^2+y^2}$ of the following Poisson equation in $\mathbb{R}^2$

$$\Delta\, u \;=\; 1$$

Note: In Analysis III you derived the representation of the Laplace operator in polar coordinates

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}.$$

Radially symmetrical means independence of the angle ϕ .

c) We are looking for the stationary (i.e. time-independent) and radially symmetrical temperature distribution in a homogeneous circular disc with inner radius $r_i \;=\; 1$, outer radius $r_a \;=\; 2$, internal temperature $T_i = 20$ and external temperature $T_a = 15$.

To this end determine a solution of $\Delta\, T(x,y) \;=\; 0$

with

$T(x,y) \;=\; 20$ for $\sqrt{x^2+y^2} = 1$ and $T(x,y) \;=\; 15$ for $\sqrt{x^2+y^2} = 2$.

Exercise 2:

a) Determine a solution of the initial value problem

$$y' + 3y + y^{\frac{3}{4}} = 0, \qquad y(0) = 1.$$

b) Show that the solution is uniquely determined in the interval $\left[0, \frac{4}{3}\ln(4)\right]$.

c) Provide a second solution in an interval $[0, L]$ with $L > \frac{4}{3}\ln(4)$.

Hand in until: 18.11.2022