Differential Equations I
for Students of Engineering Sciences

Sheet 2, Homework

Exercise 1:

a) Consider the differential equation

$$y'(x) = f(\alpha x + \beta y(x) + \gamma)$$

with $\alpha, \beta, \gamma \in \mathbb{R}$ and $\alpha + \beta f(\alpha x + \beta y(x) + \gamma) \neq 0$,

show that with the help of the substitution

$$u(x) := \alpha x + \beta y(x) + \gamma$$

it can be transformed into a differential equation with separable variables.

b) Compute the solution of the initial value problem

$$y' = \exp(x - 2y) + 0.5, \qquad y(0) = 0.$$

c) Check the solution obtained in Part b) by inserting it into the differential equation.

Exercise 2:
Determine which of the following differential equations are separable, linear, Bernoulli or a similarity differential equation. If applicable, determine the respective substitution which transforms the differential equation into a separable variables or a linear differential equation.
Which are the new differential equations obtained by the substitutions?

Note: You do not have to solve the differential equations, but you can do so if you like to!

a) $(1 + e^{2t})y' = -2e^{2t}y$

b) $\cos(t)y' + \sin(t)y = -\cos^2(t)y$

c) $y - \frac{1}{t} - \frac{1}{y}y' = 0$

d) $y' = 2t(2t^2 y^2 - 1)y$

e) $y - ty' = \dfrac{t^3}{y^2}$

Hand in until: 01.11.- 04.11.2022