Fachbereich Mathematik der Universität Hamburg **Winter Term 2022/23**
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Differential Equations I for Students of Engineering Sciences

Sheet 1, Exercise class

Exercise:

The position of a pendulum (for example a cylinder hanging on a spring) oscillating in a viscous fluid can be described by a mathematical equation. We denote by $y(t)$ the displacement from the equilibrium position. In order to simplify the model, we may assume that only:

- the reaction force, proportional to the displacement $F(t) = -D \cdot y(t)$, $D \geq 0$ and
- the damping force, proportional to the velocity $\tilde{F}(t) = -\mu \cdot y'(t)$, $\mu \geq 0$

act on the cylinder.
Newton's law of motion states that:

$$\text{mass (m)} \cdot \text{acceleration } (y''(t)) = \sum \text{ of all acting forces.}$$

a) Which differential equation would thus describe a damped pendulum?

b) Which is the order of the differential equation of part a)?

c) What is the differential equation from a) in case of an undamped motion ($\mu = 0$) of a cylinder of mass $m = 50\,kg$ when $D = 200\,N/m$?

   (i) Show that the function $c_1 \sin(2t) + c_2 \cos(2t)$ with arbitrary real constants $c_1\,, c_2$ solves this differential equation.

   (ii) Which solution(s) do we get when setting as initial speed
   $y'(0) = 0\ m/s$?
   Can you determine the position of the cylinder at a given time (for example $t = 10$)?

   (iii) Which solution(s) do we get if we set additionally an initial displacement of
   $y(0) = 0.5\ m$?

d) Let us now look for the solutions of the differential equation

$$200 \cdot y''(t) = -40 \cdot y'(t) - 202 \cdot y(t)\,.$$

   With the help of the ansatz $y(t) = ke^{\lambda t}$, $k,\ \lambda \in \mathbb{C}$ constant, determine solutions of the differential equation.

Dates: 17.10.-21.10.2022