Differential Equations I
for Students of Engineering Sciences

Sheet 1, Homework

(Review of selected topics of Mathematics II)

Exercise 1:
Determine all eigenvalues, eigenvectors and if appropriate generalized eigenvectors of the matrix

$$A = \begin{pmatrix} 5 & 1 & 0 & 0 \\ 0 & 5 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix}.$$

Exercise 2: *Please use Laplace expansion along rows or columns. Do not work solely with the rule of Sarrus!*

a) Let A be a real $n \times n$ matrix and $\lambda = a + ib$ with $a, b \in \mathbb{R}$ and $i^2 = -1$ an eigenvalue of A with corresponding eigenvector $\boldsymbol{v}$. Show that $\bar{\lambda} = a - ib$ is an eigenvalue of A with corresponding eigenvector $\bar{\boldsymbol{v}}$.

b) Determine all eigenvalues and eigenvectors of the matrix

$$B = \begin{pmatrix} 2 & 0 & 3 \\ 2 & 1 & 2 \\ -3 & 0 & 2 \end{pmatrix}.$$

Hand in until: 17.10.-21.10.2022