**Mathematics Department**
Prof. Dr. J. Behrens

**WiSe 2022/2023**

# Mathematics III Exam
# (Module: Differential Equations I)

**March 6, 2023**

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

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| **BP:** | AIW | BU | BV | CI<br>CS | ET | EUT | GES | IN<br>IIW | LUM | MB | MTB<br>MEC | SB | VT | |

I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

(Signature)

| **Exercise** | **Points** | **Evaluater** |
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| **1** | | |
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**Exercise 1: (3+4 points)**

Solve the following initial value problems

a)
$$y'(x) = \frac{1+\cos(x)}{(y(x))^2} \qquad \text{for } x > 0, \qquad y(0) = 3.$$

b)
$$x^2 y''(x) - xy'(x) - 8y(x) = 0 \qquad \text{for } x > 1, \qquad y(1) = 0,\ y'(1) = 6.$$

**Exercise 2: (4 points)**

Compute a real fundamental system of the solution space of the following system of differential equations and provide the real-valued general solution.

$$\boldsymbol{y}'(t) = \begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix} \boldsymbol{y}(t)\,.$$

**Exercise 3: (5 points)**

Consider the following fourth-order differential equation

$$y^{(4)}(t) + a_3 y'''(t) + a_2 y''(t) + a_1 y'(t) + a_0 y(t) = 0 \tag{1}$$

with real coefficients $a_0, a_1, a_2, a_3 \in \mathbb{R}$. For each of the following sets of functions, examine whether (with suitable coefficients $a_0, a_1, a_2, a_3 \in \mathbb{R}$) they could constitute a fundamental system for the solution space of the differential equation.

Justify your answers.

a) $M_1 := \{y_1(t) = e^t,\ y_2(t) = e^{5t},\ y_3(t) = e^{9t}\}$.

b) $M_2 := \{y_1(t) = e^t,\ y_2(t) = e^{it},\ y_3(t) = e^{2t},\ y_4(t) = e^{2it}\}$.

c) $M_3 := \{y_1(t) = 1,\ y_2(t) = t,\ y_3(t) = e^{2t},\ y_4(t) = e^{-2t}\}$.

d) $M_4 := \{y_1(t) = e^t,\ y_2(t) = \sin(2t),\ y_3(t) = e^{-2it},\ y_4(t) = e^{2it}\}$.

**Exercise 4: (4 points)**

Without knowing the number $\gamma \in \mathbb{R}$, decide for each of the following matrices $\boldsymbol{A}_k, k = 1, 2, 3$ whether the zero solution is a stable or an unstable stationary point (equilibrium point) of the differential equation system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_k \boldsymbol{x}(t)$.

Justify your answers.

**i)** $\boldsymbol{A}_1 = \begin{pmatrix} -2 & \gamma & 0 \\ 0 & 0 & \gamma \\ 0 & 0 & -2 \end{pmatrix}$, **ii)** $\boldsymbol{A}_2 = \begin{pmatrix} \gamma & -1 & 0 \\ 1 & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$, **iii)** $\boldsymbol{A}_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -\gamma \\ 0 & \gamma & 0 \end{pmatrix}$.