Typ	DGL	Lösung/Subst	ggf. neue DGL
separierbar	$\dot{y}(t) = h(t) \cdot g(y)$	$\int \frac{dy}{g(y)} = \int h(t)dt$	
Ähnlichkeits DGL	$\dot{y}(t) = f\left(\frac{y(t)}{t}\right)$	$u(t) := \frac{y(t)}{t}$	$u' = \frac{f(u) - u}{t}$ separierbar
Ähnlichkeits DGL	$\dot{y}(t) = f(at + by(t) + c)$	$u := at + by(t) + c$	$\dot{u} = a + bf(u)$ separierbar
Lineare homogen Lineare inhomogen	$\dot{y}_h(t) = -a(t)y_h^{1}(t)$ $\dot{y}(t) = -a(t)y^{1}(t) + h(t)$	$\int \frac{dy_h}{y_h} = -\int a(t)dt$ $y = c \cdot y_h + y_p$	separierbar y_p partikuläre Lösung Ansatz $y_p = c(t)y_h$ Bedingung: $\dot{c}(t)y_h = h(t)$
Bernoullische DGL	$\dot{y} + a(t)y + b(t)y^{\alpha} = 0$	$u := y^{1-\alpha}$ $\alpha \neq 0, 1$	$\dot{u} + (1-\alpha)a \cdot u = (\alpha - 1)b$ linear
Riccatische DGL	$\dot{y} + a(t)y + b(t)y^2 = c(t)$ b, c nicht ident. 0	$u := \frac{1}{y - y_p}$	$\dot{u} - [a + 2b \cdot y_p] \cdot u = b$ linear
Eulersche	$a_2t^2\ddot{y} + a_1t\dot{y} + a_0y = 0$	Ansatz $y(t) = t^r$	$a_2r^2 + (a_1 - a_2)r + a_0 = 0$ quadratisches Polynom

Beispiele:

Typ	DGL	Substitution	ggf. neue Diff.gleichung
Ähnlichk.	$x^2y' = y^2 + x^2e^{\frac{x}{y}}$ $y' = (\frac{y}{x})^2 + \exp(x/y) = u^2 + \exp(u^{-1}) = f(u)$	$u = y/x$	$u' = \frac{f(u)-u}{x} = \frac{u^2 + e^{1/u} - u}{x}$
separierbar	$\cos^2(x)y' = y^3$ $y' = y^3 \cdot \frac{1}{\cos^2(x)}$	—	
Riccati	$\dot{y} \underbrace{- \frac{2}{t}}_{a} y + \underbrace{\frac{1}{t^2}}_{b} y^2 = \underbrace{4t^2}_{c}$	$u = \frac{1}{y - y_p}$ bei gegebenem y_p z.B. $y_p = 2t^2$	$u' - [a + 2b y_p] u = b$ $u' - [-\frac{2}{t} + \frac{2}{t^2} 2t^2] u = 1/t^2$ ⊛
Bernoulli	$\cancel{4}\dot{y} + \frac{\overset{1}{\cancel{4}}}{t}y + \frac{1}{4}y^{\overset{\alpha}{3}} = 0$	$u = y^{1-\alpha} = y^{-2}$	$\dot{u} + (1-\alpha) a \cdot u = (\alpha - 1) b$ $\dot{u} + (-2)\frac{1}{t} u = 2 \cdot 1/4$
Ähnlichk.	$y' = \cos(\overset{a}{2x} \overset{b}{- 3y})$ $= f(u)$	$u = 2x - 3y$	$u' = a + b f(u)$ $= 2 - 3\cos(u)$
Linear	$\dot{y} + \sin(t)y = \cos(t)$	—	

⊛ Zur Kontrolle $\quad u_h = c\frac{e^{4t}}{t^2} \quad u_p = -\frac{1}{4t^2} \quad y = y_p + \frac{1}{u}$

$$y(t) = 2t^2 + \frac{4t^2}{ke^{4t} - 1} = 2t^2 \left[\frac{ke^{4t} + 1}{ke^{4t} - 1}\right]$$

Beispiel) $y + \frac{e^t}{y^2} + 3t \cdot y' = 0 \qquad t \geq 0.5, \quad y(\ln(2)) = 0$

Sortieren: $y' + \underbrace{\frac{1}{3t}}_{a} y^1 + \underbrace{\frac{e^t}{3t}}_{b} y^{-2} = 0$ $/\alpha$

Bernoullische DGL mit : $\alpha = -2.$

Substitution $u := y^{1-\alpha} = y^3 \iff y = \sqrt[3]{u}$

liefert

$u' + \underbrace{(1-\alpha)}_{3}\underbrace{a}_{\frac{1}{3t}} \cdot u = \underbrace{(\alpha-1)}_{-3}\underbrace{b}_{\frac{e^t}{3t}} \implies u' + \frac{1}{t}u = -\frac{e^t}{t} = h(t)$

Lineare DGL mit zug. homogener Gleichung:

$u_h' = -\frac{1}{t} u_h$

Lösung : Raten $(t^k)' = \frac{k}{t} \cdot t^k$ oder Separation liefert $\boxed{u_h = \frac{c}{t}}$

$u_p = \frac{c(t)}{t} \xrightarrow{\text{Dgl}} \frac{\dot{c}(t)}{t} = h(t) = -\frac{e^t}{t} \implies \dot{c}(t) = -e^t \implies c(t) = -e^t$ (f.u.)

z. B. $\boxed{u_p(t) = -e^t/t}$ zusammen $u = u_h + u_p = \frac{1}{t}(c - e^t)$

$y(t) = \sqrt[3]{\frac{1}{t}(c - e^t)}$