Analysis III for Engineering Students Work sheet 7

Exercise 1:

a) Let \boldsymbol{f} be the vector field $\boldsymbol{f}(x,y) = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$, \mathbf{c}_1 be the curve with the parametrization

$$\mathbf{c}_1(t) = (t, \sin(t))$$
 $t \in [0, \pi]$

and c_2 be the mathematically positively oriented edge of the rectangle

$$R = \{(x,y) : x \in [0,1], y \in [0,2]\} = [0,1] \times [0,2]$$

- (i) Does vector field f have a potential?
- (ii) For i = 1, 2 compute the line integrals

$$\int_{\mathbf{c}_{i}} \mathbf{f}(x,y) \, d(x,y) \, \cdot$$

- (iii) Compute the flow of \boldsymbol{f} through R.
- b) Let \tilde{f} be the vector field $\tilde{f}(x,y) = \begin{pmatrix} x^2 y^3 \\ y^2 + x^3 \end{pmatrix}$, \mathbf{c}_2 be defined as above and $\mathbf{c}_3(t) = (1, t^2)$ $t \in [0, 3].$

$$\int_{\mathbf{c}_2} \tilde{\boldsymbol{f}}(x,y) d(x,y), \qquad \int_{\mathbf{c}_3} \tilde{\boldsymbol{f}}(x,y) d(x,y) \cdot \mathbf{c}_3$$

Exercise 2) (5 + 1 + 3 + 1 Points)

Given are

$$K := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : 0 \le x^2 + y^2 + z^2 \le 16, \ y \ge 0 \right\} ,$$

and the vector field

$$\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3, \ \boldsymbol{f}(x, y, z) = \begin{pmatrix} x + y^2 \\ 2y \\ 3z + x^2 \end{pmatrix}.$$

- a) Compute $\int_{K} \operatorname{div} \boldsymbol{f}(x, y, z) d(x, y, z)$.
- b) K is bounded by a flat surface W and a curved surface $M\,.$ Provide the parametrization of $W\,.$
- c) Compute the flow of \boldsymbol{f} through W, i.e

$$\int_W \boldsymbol{f} \cdot dO$$
 .

d) According to a) and c) how large is the flow through the curved part of the edges of $K\,,$ i.e

$$\int_M \boldsymbol{f} \cdot dO?$$

Classes: 27.01–31.01.25