

Analysis III

for Engineering Students

Homework sheet 7

Exercise 1:

Given the vector fields $\mathbf{f}, \mathbf{g} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$,

$$\mathbf{f} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2xz \\ -2yz \\ x^2 - y^2 \end{pmatrix} \quad \text{and} \quad \mathbf{g} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^2 + z \\ y^2 z + z^3 \\ -y \end{pmatrix}$$

a) compute the potentials of $\mathbf{f}$ and $\mathbf{g}$, if it is possible.

b) with

$$\mathbf{c} : [0, \frac{\pi}{6}] \rightarrow \mathbb{R}^3, \quad \mathbf{c}(t) = \begin{pmatrix} t \\ \cos(3t) \\ \sin(3t) \end{pmatrix}.$$

compute the line integrals

$$\int_{\mathbf{c}} \mathbf{f} d\mathbf{x}, \quad \text{and} \quad \int_{\mathbf{c}} \mathbf{g} d\mathbf{x}.$$

Exercise 2:

Given the body $K := \{ \mathbf{x} \in \mathbb{R}^3 \mid x^2 + y^2 \leq 4, 0 \leq z \leq 5 - x + y, \}$

and the vector field $\mathbf{f}(\mathbf{x}) := (xz, yz, xyz)^T$

a) sketch the body K and provide the parametrizations for the three smooth surfaces F_1 , F_2 and F_3 , which bound K .

b) compute the volume integral $\int_K \operatorname{div} \mathbf{f} d\mathbf{x}$.

c) compute the flow of $\mathbf{f}$ through the surfaces F_1 , F_2 and F_3

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