

Analysis III for Engineering Students

Work sheet 6

Exercise 1:

a) Given the body $K \subset \mathbb{R}^3$

$$K = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid x \in [1, 3], \ 1 - x \leq y \leq 2 + x, \ x^2 + y^2 - 1 \leq z \leq x^2 + y^2 + 1 \right\},$$

and the vector field

$$\mathbf{f} : \mathbb{R}^3 \rightarrow \mathbb{R}^3, \quad \mathbf{f}(x, y, z) = \begin{pmatrix} yz + y \\ x(z + 1) + y \\ y(z + 2) + x \end{pmatrix}.$$

Compute

$$\int_K \operatorname{div}(\mathbf{f}(x, y, z)) \, d(x, y, z).$$

b) Compute the integral

$$\int_D (1 - x^2) d(x, y)$$

over the annulus

$$D := \{(x, y)^T \in \mathbb{R}^2; \ 1 \leq x^2 + y^2 \leq 4\}.$$

Hint: $\cos^2 \phi = \frac{1}{2} (\cos(2\phi) + 1).$

Exercise 2:

Let $K \subset \mathbb{R}^3$ be the cone $K = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 0 \leq x^2 + y^2 \leq 1, \ 0 \leq z \leq 4 - 4\sqrt{x^2 + y^2} \right\}$ with constant density $\rho = 2$.

a) Compute the mass of the cone.

b) Compute the moment of inertia of the cone with respect to the z -axis using integration.

Classes: 13.01–17.01.22