**Fachbereich Mathematik der Universität Hamburg** **WiSe 2024/25**
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# Analysis III for Engineering Students

## Sheet 6 , Homework

**Exercise 1:**

a) Let

$$D := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \,:\, \frac{y^2}{2} - 2 \,\leq\, x \,\leq\, 4 - y^2 \right\}.$$

Sketch the set $D$ and determine the center of mass of $D$ with uniform mass density (mass/area) $\rho = 2$.

Hint: It holds

**Mass:** $M = \int_D \rho(\boldsymbol{x})\, d\boldsymbol{x}$

**Center of mass:** $X_s = \frac{1}{M} \int_D \rho(\boldsymbol{x})\; \boldsymbol{x}\; d\boldsymbol{x}$ (componentwise)

b) Let $K := \left\{ (x,y,z)^T \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1,\, z \geq 0 \right\}$. Compute

$$\int_K (y^2 - x^2)\, d(x,y,z)$$

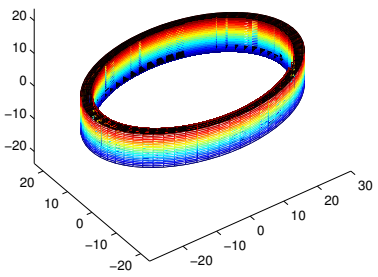
*Hint:*
- *Using* **spherical coordinates** *might be helpful.*
- $\cos(2t) = \cos^2(t) - \sin^2(t)$.

**Exercise 2:**

Given is the elliptical pipe section

$$R \subset \mathbb{R}^3, \quad R : 81 \leq \left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 \leq 100, \quad -5 \leq z \leq 5.$$

with constant density $\rho$.



Compute the volume, mass and moment of inertia of the pipe section with respect to the $y-$ axis using integration. Use elliptical cylindrical coordinates

$$x = 3r\cos(\varphi)\,,\; y = 2r\sin(\varphi)\,,\; z = z\,.$$

**Hint:**

$\cos^2(\phi) = \frac{\cos(2\phi)+1}{2}\,.$

Since we do not use a calculator, there is no need to calculate the precise final value. It is sufficient to insert the integration limits into the calculated antiderivatives only.

**Hand in until:** 17.01.25