Fachbereich Mathematik der Universität Hamburg **WiSe 2024/25**
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Analysis III for Engineering Students

Work sheet 4

Exercise 1: Determine the stationary points of the following functions and check whether they are minima, maxima or saddle points:

a) $f(\boldsymbol{x}) := \boldsymbol{x}^T \boldsymbol{A}\, \boldsymbol{x} + \boldsymbol{b}^T \boldsymbol{x} + c$ with

$$\boldsymbol{x} := \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2, \quad \boldsymbol{A} := \begin{pmatrix} 9 & -3 \\ -3 & 4 \end{pmatrix}, \quad \boldsymbol{b} := \begin{pmatrix} 6 \\ -8 \end{pmatrix}, \quad c = 24,$$

b)

$$g : \mathbb{R}^2 \to \mathbb{R}, \quad g(x,y) := x^3 + y^3 - 27xy + 25.$$

Exercise 2:

Let $g(x,y) := y^2 \cdot x - y \cdot \exp(x+y) + 2$.

a) Show that $g(x,y) = 0$ implicitly defines a function $y(x)$ in the neighbourhood of $P_0 = (-1,1)$, i.e. the following holds locally

$$g(x,y) = 0 \Longrightarrow y = f(x), \quad f(-1) = 1.$$

b) Compute the first-order Taylor polynomial of f from part a) centered at $x_0 = -1$.

c) Calculate $f'(-1)$ using implicit differentiation.

d) The equation $g(x,y) = y^2 \cdot x - y \cdot \exp(x+y) + 2 = 0$ implicitly describes a curve in $\mathbb{R}^2$.

Why is it impossible for P_0 to be a singular point on the curve?

Check whether the curve has a horizontal or vertical tangent at P_0.

Classes: 02.-06.12.24