Fachbereich Mathematik der Universität Hamburg **WiSe 2024/25**
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Analysis III for Engineering Students

Sheet 4, Homework

Exercise 1:

The equation $g(x, y) = x^4 - x^2 + y^2 = 0$ implicitly describes a curve in $\mathbb{R}^2$.

Determine the symmetries of this curve, its singular points (+ Classification) and the curve points with horizontal or vertical tangents.

Exercise 2: Given a function $g(x, y) := x^4 + y^4 + 8xy = 0$.

a) (i) Using the implicit function theorem show that $g(x, y)$ can be solved for y near the point $(x_0, y_0)^T := (2, -2)^T$. This means that there exists a function $f(x)$ with $f(2) = -2$, such that in some neighborhood of x_0 and y_0 the following equivalence holds

$$g(x, y) = 0 \iff y = f(x).$$

(ii) Compute the first-order Taylor polynomial of the function f from part (i) centered at a point $x_0 = 2$.

(iii) Compute the second-order Taylor polynomial of the function f from part (i) centered at a point $x_0 = 2$.

b) Using the Implicit Function Theorem show that the solution set of

$$g(x, y, z) := (x^2 - 2e^{xy})z + 2 = 0$$

in a neighborhood of the point $P_0 := (x_0, y_0, z_0)^T := (0, 1, 1)^T$ can be solved for x. This means that there is a function $f(y, z)$ with $f(1, 1) = 0$ such that in a neighborhood of x_0, y_0, z_0 it holds

$$g(x, y, z) = 0 \iff x = f(y, z).$$

For which other variable(s) can one solve the problem using the Implicit Function Theorem?

Hand in until: 06.12.24