Fachbereich Mathematik der Universität Hamburg **WiSe 2024/25**
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Analysis III
for Engineering Students
Work sheet 3

Exercise 1: Given a function

$$f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) := x \cdot y^2 + \cos(x+y) + 2y + 3\,.$$

a) Compute the second degree Taylor polynomial T_2 of f at $(x_0, y_0) = (0,0)$.

b) Show that for the remainder $R_2(x,y) = f(x,y) - T_2(x,y)$ in the area $|x| \le 0.2\,,\ |y| \le 0.2$ the following estimate holds:

$$|f(x,y) - T_2(x,y)| \le \frac{4}{100}\,.$$

c) What is the third-degree Taylor polynomial T_3 of f ar $(x_0, y_0) = (0,0)$?
Show that for all $(x,y) \in \mathbb{R}^2\,,\ |x| \le 0.2$ and $|y| \le 0.2$

$$|f(x,y) - T_3(x,y)| \le \frac{2}{1000}\,.$$

holds.

Exercise 2:

a) Describe the following subsets of $\mathbb{R}^2$ and $\mathbb{R}^3$ in words and describe them using polar, cylindrical or spherical coordinates.

$$M_1 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \, : \, x^2 + y^2 \leq 4 \right\},$$

$$M_2 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \, : \, x^2 + y^2 \leq 4, \, y \geq 0 \right\},$$

$$M_3 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \, : \, x^2 + y^2 \leq 4, \, x \geq 0 \right\},$$

$$M_4 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \; x^2 + y^2 \leq 4, 0 \leq z \leq 5 \right\},$$

$$M_5 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \; x^2 + y^2 + z^2 \leq 4 \right\},$$

$$M_6 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \; x^2 + y^2 + z^2 \leq 4, \, y \geq 0 \right\},$$

$$M_7 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \; x^2 + y^2 + z^2 \leq 4, \, z \geq 0 \right\}.$$

b) Describe the boundaries of the sets from a) using polar, cylindrical or spherical coordinates.

Classes: 18.–22.11.24