

Analysis III for Engineering Students Sheet 3, Homework

Exercise 1: (6+4 Points) Let

$$f(x, y, z) = 2 + xz + y^2 + e^x y^2 \cos(z).$$

- a) Compute the second degree Taylor polynomial T_2 of f at $\mathbf{x}_0 = (x_0, y_0, z_0)^T := (0, 1, \pi)^T$.
- b) Show that for the remainder $R_2(x, y, z) = f(x, y, z) - T_2(x, y, z)$ the following estimate holds

$$|R_2(x, y, z)| \leq 0.02 \quad \forall \mathbf{x} = (x, y, z)^T \in \mathbb{R}^3 : \|\mathbf{x} - \mathbf{x}_0\|_\infty \leq 0.1.$$

Exercise 2:

Note: To solve this problem, you do not need to calculate a single derivative exactly!

Calculate the second-degree Taylor polynomial T_2 for the function

$$f(x, y) = xy + \cos(x) e^y + \sin\left(\frac{x+y}{2}\right)$$

at $\mathbf{x}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and show that for all

$$(x, y)^T \in \mathbb{R}^2 \quad \text{with} \quad |x| \leq 0.15, \quad |y| \leq 0.2$$

the following estimate holds

$$|R_2(x, y; \mathbf{x}_0)| := |f(x, y) - T_2(x, y; \mathbf{x}_0)| \leq 0.05.$$

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Hand in until: 22.11.24