Analysis III
for Engineering Students
Work sheet 1

Exercise 1:

Let

$$f : \mathbb{R}^2 \to \mathbb{R}, \qquad f(x,y) = \cos(2x - 3y) + x^3 - y^3 + 2y^2.$$

a) Find all first, second and third order partial derivatives of f.

b) Determine $\operatorname{grad} f(x,y)$, $\nabla f(x,y)$ and $\Delta f(x,y)$.

Exercise 2: Consider the following sets

$$M_1 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R},\ x^2 + y^2 \le 1 \right\},$$

$$M_2 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R},\ x^2 + y^2 < 4 \right\},$$

$$M_3 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R},\ 1 \le x^2 + y^2 < 4 \right\},$$

$$M_4 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R},\ x^2 + y^2 \le 1 \right\},$$

$$M_5 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R},\ x^2 + y^2 + z^2 < 1 \right\},$$

$$M_6 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 : (x,y) \cdot (1,2)^T = 1 \right\},$$

$$M_7 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : (x,y,z) \cdot (1,2,1)^T < 1 \right\},$$

$$M_8 := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R},\ z = x^2 + y^2 \right\}.$$

$$M_9 := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R},\ (x+3)^2 + y^2 \le 1 \right\} \cup \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x, y \in \mathbb{R},\ (x-3)^2 + y^2 \le 1 \right\}.$$

a) Which are the boundary points of $M_1, \ldots, M_9$?

b) Decide for each set $M_1, \ldots, M_9$ if it is closed, open or neither closed nor open.

c) Which of the sets $M_1, \ldots, M_9$ are bounded?

d) Which sets $M_1, \ldots, M_9$ are connected? Which are convex?

Classes: 21.–24.10.24