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Advice on the exam Analysis III

The files provided online are only intended to support the corresponding course. These documents are incomplete without the additional explanations given during the session (e.g. essential requirements are often missing). Typing or spelling mistakes that are noticed in time will only be announced verbally during the course. Corrections will not necessarily be made online.

The listing of important topics does NOT imply the exclusion of other topics for the exam.

Absolutely necessary tools:

- Confident skills in partial derivation,
- $\nabla f = \textbf{grad}\, f^T =$ Vector of first derivatives,
- $\nabla^2 \boldsymbol{f} = H\boldsymbol{f} =$ Matrix of second derivatives,
- graph of elementary functions and their derivatives,
- compute eigenvalues, specifically their signs,
- **curl** $\boldsymbol{f}$, div $\boldsymbol{f} =$ divergence,
- scalar (dot) product, cross product, Jacobian Matrix
- elementary integration, very simple substitution ($\cos(k\phi)$ etc.), partial integration.

- $\cos^2(\alpha) + \sin^2(\alpha) = 1$.

Trigonometric identities for $\cos^2(\alpha)$, $\sin^2(\alpha)$, $\cos(\alpha)\sin(\alpha)$, $\cos^3(\alpha)$, $\sin^3(\alpha)$ are provided if necessary.

Abbreviations

wz: tool

xxx: Important type of exercise, you can gain/loose a lot of points.

x: May occur. But in most cases as a little exercise, few points.

$\emptyset$: Not suitable as an exam task.

Hints:

- **Pay attention to the number of points!**

  A task with 1-2 points should not keep you busy for 15 minutes.

  The answer to a task with with ≥ 4 points will not be finished with just one line.

- If your calculation leads to a complicated number/term for example

$$26 \pm \sqrt{26 - \frac{18}{2}}$$

  – this is ok if it occurs at the end of your solution (e.g. if you only need the sign of that number or if it is the final result of an integration). But not if this number is needed in further calculations. In this case you should check the steps leading to this term. It might be that the true result is the simpler number

$$25 \pm \sqrt{25 - \frac{18}{2}} = 25 \pm \sqrt{16}.$$

Top Topics of the last exams

(In the order of treatment in the lecture)

- **Taylor polynomial with error estimation**

  Exercises: Sheet 3: P1, H1, H2

- **Min/Max without constraints**

  – Find candidates: **grad** $f = \mathbf{0}$
  – Classification: Eigenvalues of the Hessian Hf

  Exercises: Sheet 4: P1

- **Min/Max with constraints**
  - admissibility, regularity condition,
  - set up Lagrangefunktion F,
  - stationary points: **grad** $F = 0$
  - compute Hessian $\boldsymbol{H}\, F$
  - Classifikation:
    * Set of admissible points compact?
    * check definiteness of Hessian.
    * If necessary compute tangential space, check definiteness of Hessian on tangential space

  EXercises: Sheet 5: P1, P2, H1

- **Multiple integrals**
  - direct calculation
  - Transformation theorem (Polar–, cylinder–, Spherical coordinates)
  - Volume, Mass, moment of inertia, flow (Gauss)

  Exercises: Sheet 7: P2, H2, Sheet 6: all exercises.

- **Line integrals**

  - Compute **curl** $\boldsymbol{f}$
  - Compute potential $\longrightarrow$ compute line integral using fundamental theorem
  - Compute line integral directly (without potentials)
  - Theorems of Green (Gauß for vector fields in $\mathbb{R}^2$)

  Exercises: Sheet 7: P1, H1

- **Surface integrals**

  - Parametrisation
  - Flow, flux, divergence theorem/Gauß theorem

  Exercises: sheets 7: P2, H2

- **Sheets 1:**

  - P1: Compute partial derivatives, **grad**, ∇ , Δ. **(wz)**

  - P2: Subsets in $\mathbb{R}^2$, $\mathbb{R}^3$, circles, rings, spheres, etc.
    Definitions: limited, closed, convex etc. **(wz)**

  - H1: Compute derivatives up to order 3 ,
    $\nabla-$operator. Theorem of Schwarz on exchangeablity! **(wz)**

  - H2: Show given function solves wave equation. **($\emptyset$)**

- **Sheets 2:**
  P1 a: show: **curl grad**$(\boldsymbol{f}) = \mathbf{0}$, $\forall \mathcal{C}^2$ functions f
  gradient fileds are curl-free. **(wz)**

  P1 b: Decide which given vector fields cannot be
  a gradient fields. **(x,wz)**

  P2: Compute **grad** f, **(wz)**

  Draw the contour lines and direction of the gradient. **(∅)**

  P2 d: Direction of the gradient versus contour lines **(∅)**

  H1: Compute Jacobi matrix and determinants. **(wz)**

  H2: : Level surfaces, directional derivatives,
  ascent or descent direction?. **(wz)**

- **Sheets 3:**

  - P1a,b: Taylor polynomial T_2 in $\mathbb{R}^2$ with error estimation **(xxx)**
  - P1c: Taylor polynomial T_3 in $\mathbb{R}^2$ with error estimation **($\emptyset$)**
  - P2: Describe sets using polar, cylindrical or spherical coordinates. **(wz)**
  - H1: Second order Taylor polynomial, $\mathbb{R}^3, \boldsymbol{x}_0 \neq \mathbf{0}$ with error estimate. (Too time-consuming in this combination, but good training for the exam)
  - H2: Determine second order Taylor polynomial, $\mathbb{R}^2$ using sine / cosine / exponential series with error estimation. **(xxx)**

- **sheets 4:**

  - P1: Extrema (Min/Max) without constraints.
    Find stationary Points
    and classify: min/max/saddle? **(xxx)**

  - P2a,b: Implicit Function Theorem $\mathbb{R}^2$
    Compute T_1in addition. **(x)**

    P2c: implicit differentiation. . **(∅)**
  - H1: Curve in $\mathbb{R}^2$. Singular points + classification,
    Points with horizontal/vertical tangents. **(x)**
  - H2a: Implicit Function Theorem $\mathbb{R}^2$
    Computation of T_1 **(x)**
    und T_2 **(∅)**

    H2b: Implicit Function Theorem $\mathbb{R}^3$. **(x)**

- **Sheets 5:**

  - P1: Extrema (Min/Max) under constraint $g(x, y) \leq 0$ in $\mathbb{R}^2$, **(xxx)**

    * P1a: Extrema in the Interior $g(x, y) < 0$,
    * P1b: Extrema on the boundary $g(x, y) = 0$,
    * P1c: Extrema in the whole set of admissible points (Compactness!)

  - P2: Extrema (Min/Max) under constraint in $\mathbb{R}^2$. **(xxx)**

    If a candidate is given, check:
    regularity condition, admissibility, necessary conditions.
    Classification: Investigate Hessian on tangential space.

  - H1: Min/Max under two constraints in $\mathbb{R}^3$. **(too much work)**

    Classification: Use Compactness.

- H2: Newton's Method. **(∅)**

- **Sheet 6:** **(Everything xxx)**

  - P1: Multiple integrals,, divergence , Theorem of Transformation, polar coordinates.

  - P2: Multiple integral, cylinder coordinates,
    mass, moment of inertia.

  - H1a: Multiple integral, cartesian, center of mass

  - H1b: Multiple integral, spherical coordinates

  - H2: Multiple integral, elliptic cylinder coordinates

    (More likely for the exam: a task on standard cylindrical coordinates instead of elliptical cylindrical coordinates)

volume, mass, moment of inertia.

– **Sheet 7:** **(Everything xxx)**

* P1a: Compute potential, compute line integral,
  flux (Green/Gauß in der Ebene)

* P1b: compute line integral directly, Green's theorem

* P2: Multiple integral, surface integral, Gauß, Spherical coordinates.

* H1: Curl, potential,
  compute line integral directly and by use of a potential.

* H2: Multiple integral, surface integral, Gauß,,
  cylinder coordinates

  (More likely for the exam: an object with two boundary surfaces instead of three)