Mathematics Department **WiSe 2024/2025**
Prof. Dr. I. Gasser

Mathematik III Exam
(Module: Analysis III)

March 4, 2025

Please mark each page with your name and your matriculation number.

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

Signature:

Exercise	**Points**	**Evaluater**
1		
2		
3		
4		

$\sum$ =

Exercise 1: (7 Points)

Determine and classify all local extrema of $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x, y) = 8x - \frac{9}{2}y$$

under the constraint

$$g(x, y) = 16x^2 + 9y^2 - 25 = 0$$

using the Lagrange multiplier rule. First check the regularity condition.

Exercise 2: (4 Points)

Given the half circular ring

$$D := \left\{(x,y)^T \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 9,\, y \geq 0\right\}$$

with mass density $\rho(x,y) = 3 - y$

compute the mass m of D.

Exercise 3: (5 Points)

a) Show that there exists no potential for the function $g : \mathbb{R}^3 \to \mathbb{R}^3$

$$g(x, y, z) = (x + \frac{1}{2}yz\,,\ -y + z\,,\ -y).$$

b) Compute the line integral

$$\int_c g(x, y, z)d(x, y, z)$$

along the curve

$$c(t) = \begin{pmatrix} t \\ \sin(\frac{t}{2}) \\ \cos(\frac{t}{2}) \end{pmatrix} \qquad c : [0\,,\ \pi] \to \mathbb{R}^3.$$

Exercise 4: (1+3 Points)

Let $\boldsymbol{f}$ be the vector field

$$\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2, \ \boldsymbol{f}(x,y) = \begin{pmatrix} xy + \tan(e^{-x^2}) \\ x^2 - \cos(e^{-y^2}) \end{pmatrix}$$

a) Compute $\mathbf{curl}\, \boldsymbol{f}(x,y)$.

b) For the mathematically positively oriented boundary ∂D of the triangle

$$D := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \, : \, 0 \leq x \leq 2, \, 0 \leq \, y \leq \, 2 - x \right\}$$

compute the line Integral $\displaystyle\int_{\partial D} \boldsymbol{f}(x,y)\, d(x,y)$.