Mathematik III Exam
(Module: Analysis III)
August 27, 2025

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

Signature:

Exercise	Points	Evaluater
1		
2		
3		

$$\sum =$$

Exercise 1: (4 Points)

Determine and classify the stationary point of the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x, y) := x^2 - 4xy + 36y^2 - 10x - 12y \,.$$

Exercise 2: **(3+3 Points)**

a) Determine a potential for the function $f : \mathbb{R}^3 \to \mathbb{R}^3$

$$\boldsymbol{f}(x, y, z) = (2xy^2 , z + 2yx^2 , y)^T.$$

b) Compute the line integral

$$\int_{\boldsymbol{c}} \boldsymbol{f}(x, y, z) d(x, y, z)$$

along the curve

$$\boldsymbol{c}(t) = \begin{pmatrix} t + 1 \\ t^2 + 2 \\ t^3 \end{pmatrix} \qquad \boldsymbol{c} : [0, 1] \to \mathbb{R}^3.$$

Exercise 3: (5+1+3+1 Points)

Consider the half ball $K := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 25 , \ z \geq 0 \right\}$

and the vector field $\boldsymbol{f}(x, y, z) = \begin{pmatrix} z^2 + x^2 \\ x^2 + y \\ 2z + 1 \end{pmatrix} .$

a) Compute the integral $\displaystyle\int_K \operatorname{div} \boldsymbol{f}(x, y, z) \, d(x, y, z) .$

   Note: Depending on the order of the variables you integrate by, the following **might** be helpful

   $$2\cos^2(\alpha) = \cos(2\alpha) + 1.$$

b) The domain K is bounded by a flat surface B and a curved surface M. Determine a parametrization of the flat surface B.

c) Compute the flow of $\boldsymbol{f}$ through B.

d) Determine the flow of $\boldsymbol{f}$ through M using the results from parts a) and c).