Department of Mathematics, University of Hamburg **WiSe 2023/24**
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Analysis III for Engineering Students

Work Sheet 7

Exercise 1:

Given a vector field $\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3$ with

$$\boldsymbol{f}(x,y,z) = \left(\sin y + 3x^2 z^2, x\cos y + \frac{1}{1+y^2}, 1 + 2x^3 z \right)^T .$$

a) Show the existence of a potential for $\boldsymbol{f}$ without calculating it.

b) Calculate a potential by successively integrating $\boldsymbol{f}$ and

c) using the fundamental theorem for line integrals.

d) Given a curve $\mathbf{c} : [0, 3\pi/2] \to \mathbb{R}^3$ with $\mathbf{c}(t) = (\cos t, 0, \sin t)^T$. Compute the line integral

$$\int_c \boldsymbol{f}(\boldsymbol{x})\, d\boldsymbol{x} .$$

e) Plot the curve $\mathbf{c}$ using the MATLAB function 'plot3'.

Exercise 2:

Given a vector field $\boldsymbol{f}(x,y,z) = \left(0, 0, z^3\right)^T$ and the body

$$H = \left\{ (x,y,z)^T \in \mathbb{R}^3 \;\middle|\; x^2 + y^2 + z^2 \le 16\,,\; 0 \le y \right\} .$$

a) Make a sketch of H.

b) Give parameterization for each of surface segments bounding H.

c) Calculate the flow of $\boldsymbol{f}$ through these boundary segments.

d) Compute the volume integral $\displaystyle\int_H \operatorname{div} \boldsymbol{f}(x,y,z)\, d(x,y,z)$.

Discussion: 29.1. - 2.2.2024