Department of Mathematics, University of Hamburg **WiSe 2023/24**
Prof. Dr. J. Struckmeier
Dr. K. Rothe, Md. T. Hassan

Analysis III for Engineering Students

Homework sheet 6

Exercise 1:

a) For the function

$$f : Q \to \mathbb{R}\,, \quad f(x,y) = 6 - 2x + 4y$$

with $Q := [0,3] \times [0,2]$ compute

(i) Riemannian upper and lower sum for the following equidistant decomposition Z of Q

$$Q_{i,j} = [x_{i-1}, x_i] \times [y_{j-1}, y_j]\,, \quad i,j = 1,\ldots,n$$

where $x_i = \dfrac{3i}{n}$ and $y_j = \dfrac{2j}{n}$

(ii) and the integral of f over Q using Fubini's theorem.

b) (i) Draw the area P enclosed by the functions $f(x) = 2x$ and $g(x) = 24 - 2x^2$ and represent it as the "normal" area.

(ii) Compute $\displaystyle\int_P x\, d(x,y)$.

Exercise 2:

Draw the half cylinder Z given by $1 \le z \le 2$, $0 \le y$ and $x^2 + y^2 \le 9$ and calculate its center of mass with the density function $\rho(x,y,z) = z$ using cylindrical coordinates.

Exercise 3:

a) For the vector field $\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2$ with $\boldsymbol{f}(x, y) = \begin{pmatrix} y + \sin x \\ xy^2 \end{pmatrix}$ calculate the integral of the curve (line integral) $\oint_{\mathbf{c}} \boldsymbol{f}(\boldsymbol{x}) d\boldsymbol{x}$.

Here $\mathbf{c}$ is the mathematically positive boundary curve of the area G enclosed by $x^2 \leq y \leq x$ with $0 \leq x \leq 1$.

b) For the vector field $\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3$ with $\boldsymbol{f}(x, y, z) = \begin{pmatrix} -z^2/2 \\ 0 \\ xz \end{pmatrix}$

calculate the line integral $\int_{\mathbf{c}} \boldsymbol{f}(\boldsymbol{x}) d\boldsymbol{x}$ with the line

$\mathbf{c} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}^3$ and $\mathbf{c}(t) = \begin{pmatrix} 2\cos^2 t \\ 2\sin t \cos t \\ 2\sin t \end{pmatrix}$.

Submission deadline: 19.1.2024