

Analysis III for Engineering Students

Work Sheet 5

Exercise 1:

Examine the implicitly given by the level set curve(s)

$$f(x, y) := y^4 - 2y^2 + x^4 - 2x^2 = 0.$$

In particular, determine

- a) the symmetries of the curve(s),
- b) the points of the curve with the horizontal and
- c) vertical tangents,
- d) the singular points of the curve and classify them,
- e) draw the level set.

Exercise 2:

Compute and classify the extrema of the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ with $f(x, y) = 4x^2 + y^2$ on the circle $x^2 + y^2 - 2x = 3$

- a) by using Lagrange multipliers method and
- b) by using polar coordinate parameterization \mathbf{c} of the circle and then solving the extreme problem $h(t) := f(\mathbf{c}(t))$.

Discussion: 18.12. - 22.12.2023